PHILIP ALLAN FOR HODDER EDUCATION

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Group 5: answers

If I were to write an exploration...

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Answers to the problem on p. 33.

Since the length of the hypotenuse of a right-angled triangle is greater than the sum of its height and base then the network of the path can be minimised further by moving the junction M to a new position on BQ as shown in Figure 7. Segments AM and CM now form the hypotenuses for right-angled triangles AQM and CQM respectively.

Let the total distance of this new network be L_R and the angle MAQ be β.

From Figure 7 we can express L_R as:

$$\begin{aligned} L_R &= BM + AM + CM. \\ &= (BQ - MQ) + 2AM \quad ; \text{since } AM = CM \\ &= 50\sqrt{3} - \frac{50}{\cos\beta}\sin\beta + 2\left(\frac{50}{\cos\beta}\right) \\ &= 50\sqrt{3} - 50\tan\beta + \frac{100}{\cos\beta} \end{aligned}$$

In order to minimise the total distance L_R, we will differentiate L_R with respect to β.

$$\frac{dL_R}{d\beta} = -\frac{50}{cos^2\beta} + \frac{100\sin\beta}{cos^2\beta}$$

By setting $\frac{dL_R}{d\beta} = 0$, we obtain the following.

$$\begin{aligned} 100\sin\beta - 50 &= 0 \\ \sin\beta &= \frac{1}{2} \\ \beta &= 30^o \end{aligned}$$

We can easily verify that the second derivative in this case is positive and that L_R has a global minimum. Substituting β with 30^o, we find that BM, AM and CM have the identical length of $\frac{100\sqrt{3}}{3}$ metres. Thus, we conclude that L_R is $3\left(\frac{100\sqrt{3}}{3}\right) = 100\sqrt{3}$ metres which is less than $50\sqrt{3} + 100$ metres in Figure 6. We also know that the angle between any pair of paths that are next to each other radiating from junction M is 120^o. The junction M is the centre of a circle with radius $\frac{100\sqrt{3}}{3}$ metres. Sites A, B and C are on this circle. Since the shortest distance between any two points in a Euclidean plane is a straight line then the network in Figure 7 is the shortest possible total distance of all possible networks connecting three sites that form an equilateral triangle.

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