Introduction

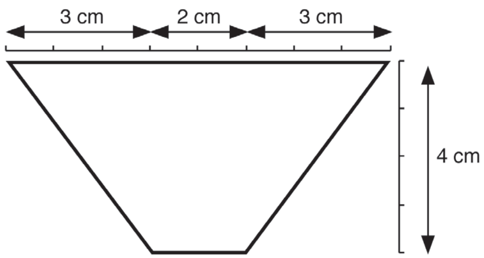
One of the main philosophies behind the Making Sense of Maths materials is the delaying of formal procedures. Rules and algorithms emerge from students' own ideas and stay connected to those ideas so that students can make sense of them. Nowhere is this more evident than in the work on area and volume.

Our experience from teaching, observing, and analysing students' work is that too often area and volume are quickly reduced to numerical procedures that, certainly in the minds of the students, bear no resemblance to anything to do with 'filling', 'shape' or 'space'. When these students are then faced with area and volume problems, their only recourse is to try and remember the procedures. Particularly for students who find mathematics difficult, this is rarely a recipe for success.

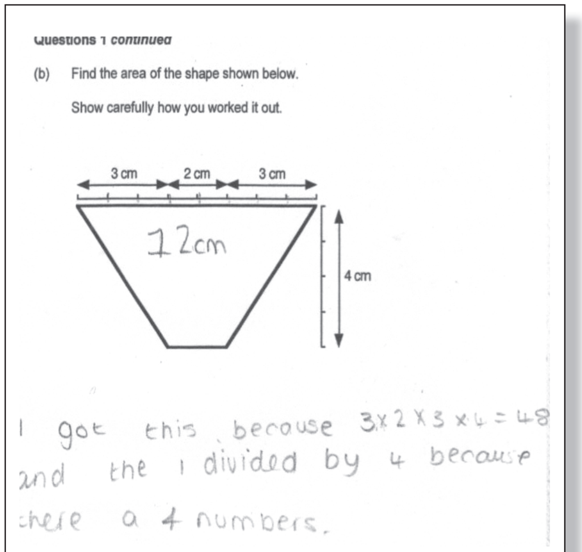
As part of the trials for these materials, students were asked to find the area of a trapezium. The students concerned had not at this stage met the formula for this shape. Some of the students had been working with our materials ('project students') and some had not ('control students'). The work of over 400 students was analysed.

When faced with the question:

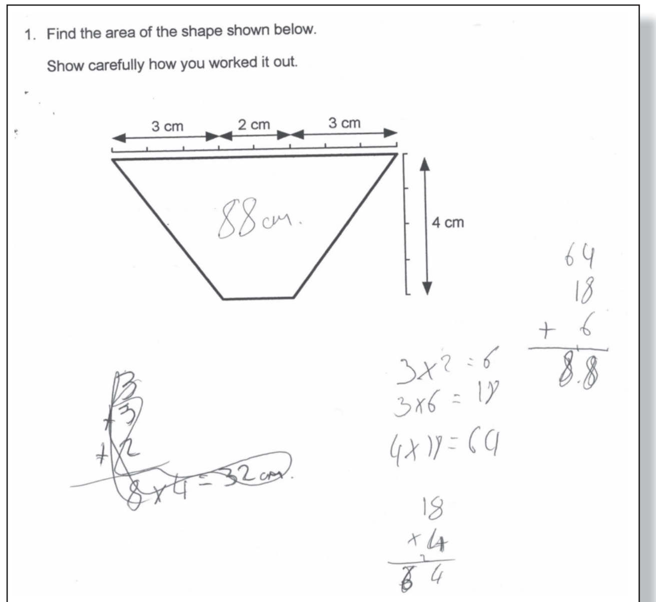
Find the area of the shape shown below. Show carefully how you worked it out.



the overwhelming majority of control students simply decided to adopt a numerical procedure, usually involving a combination of multiplying and dividing. Some typical responses from these students are reproduced here:



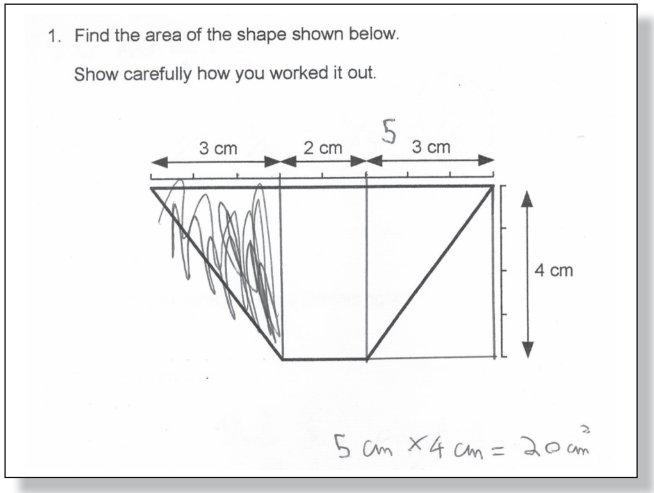
A control student making no sense of the problem, but simply attempting to 'do something with the numbers'.



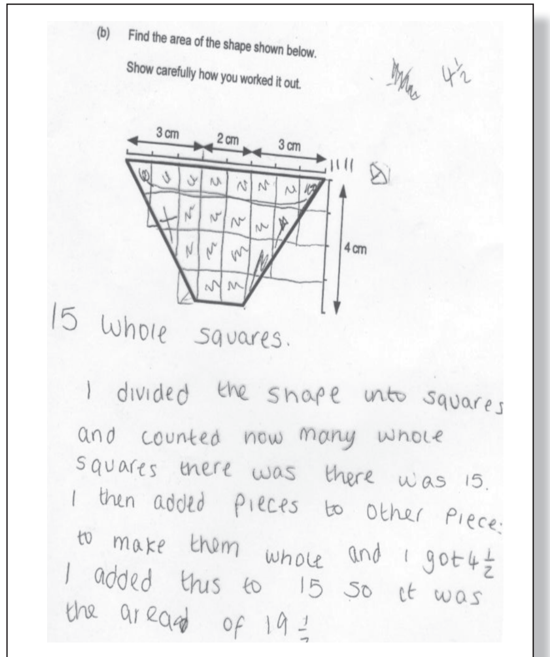
A control student attempting two different multiplication approaches, with no attempt to make sense of the problem from an area perspective.

Perhaps most telling of all was the student who didn't actually attempt the question but simply wrote, 'I can't remember how to do area like this, but I do remember it's got something to do with timesing'!

When the same question was given to project students, a very different picture emerged. Of course, there were still plenty of these students who multiplied numbers together, but far more adopted other strategies. These ranged from drawing and counting squares, to splitting into triangles and rectangles, to 'reallocating' a triangle. Altogether, over 60% of project students adopted such strategies, though of course not all got the final answer correct.



A project student using a 'reallocating' strategy to correctly solve the problem.



A lower ability project student making sense of the problem despite achieving an incorrect answer.

This second example is interesting for a number of reasons.

Teachers have reported that students sometimes feel that drawing squares (or drawing anything!) is 'babyish' and not 'proper' maths, and are resistant to this. Yet 'fitting in' squares and cubes is the basis of all work on area and volume, and it is important that teachers resist students' (and their own!) desires for formulas and 'quick ways' to arrive at answers. It was not unusual in project classes, even in revision lessons, to hear teachers asking questions such as, 'Why have you multiplied the numbers together?', 'How does your calculation count the squares?', 'How are you stacking the cubes?' etc.

We obviously want students to have procedures for solving problems, but not at the expense of understanding. We want the procedures to stay connected to what students understand so that, if they forget, they have something to fall back on.

In trials, teachers have come to see their role as one that values student intuitions and supports mathematical development whilst keeping hold of the meaning behind the symbols and procedures. They have found that they sometimes spend more time on topics but do not need to revisit topics as much as in the past. Also, when they do, they do not have to start 'from the beginning' as much.

We hope that this approach 'makes sense' to you, and that your students experience success through working in this way.