Review exercise R.2

1. (i) Plot the points $z_1 = 3 + 5\text{i}$ and $z_2 = -4 + \text{i}$ on an Argand diagram.
   (ii) Plot the points $-z_1$ and $-z_2$ and describe the geometrical connection between these points and the original points.
   (iii) Plot the points z_1^* and z_2^* and describe the geometrical connection between these points and the original points.

2. Given that $z = 3 - \text{i}$, represent the following by points on a single Argand diagram.
   (i) z (ii) $-z$ (iii) z^* (iv) $-z^*$
   (v) $\text{i}z$ (vi) $-\text{i}z$ (vii) $\text{i}z^*$ (viii) $(\text{i}z)^*$

3. Given that $z = 2 + 3\text{i}$ and $w = 1 - 2\text{i}$, represent the following complex numbers on an Argand diagram.
   (i) z (ii) w (iii) $z + w$
   (iv) $z - w$ (v) $w - z$

4. Given that $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}\text{i}$:
   (i) (a) Calculate z^0, z^1 and z^2.
   (b) Plot the points A, B and C representing z^0, z^1 and z^2 on an Argand diagram.
   (ii) By finding the lengths AB, AC and BC, show that triangle ABC is equilateral.

5. (i) Simplify the complex number $z_1 = \frac{2\text{i}}{3 - 7\text{i}}$ and find the complex number $z_2 = \text{i}z_1$ where $z_1 = \frac{2\text{i}}{3 - 7\text{i}}$.
   (ii) Plot the points A and B representing the complex numbers $z_1 = \frac{2\text{i}}{3 - 7\text{i}}$ and $z_2 = \text{i}z_1$ on an Argand diagram.
   (iii) Describe the geometrical relationship between z_1 and z_2.
   (iv) Show that the points O, A and B form an isosceles triangle.

KEY POINTS

1. Complex numbers are of the form $z = x + y\text{i}$ with $\text{i}^2 = -1$.
   x is called the real part, $\text{Re}(z)$, and y is called the imaginary part, $\text{Im}(z)$.
2. The conjugate of $z = x + \text{i}y$ is $z^* = x - y\text{i}$.
3. To add or subtract complex numbers, add or subtract the real and imaginary parts separately.
   $$(x_1 + y_1\text{i}) \pm (x_2 + y_2\text{i}) = (x_1 \pm x_2) + (y_1 \pm y_2)\text{i}$$
4. Multiplication: Expand the brackets then simplify using the fact that $\text{i}^2 = -1$
5. Division: Write as a fraction, then multiply top and bottom by the conjugate of the bottom and simplify the answer.
6. Two complex numbers $z_1 = x_1 + y_1\text{i}$ and $z_2 = x_2 + y_2\text{i}$ are equal only if $x_1 = x_2$ and $y_1 = y_2$.
7. The complex number $z = x + \text{i}y$ can be represented geometrically as the point (x, y). This is known as an Argand diagram.