Introduction

The contexts chosen in this section are designed to naturally introduce and promote the use of certain strategies (or models). The strategies (or models) associated with these contexts are as follows:

- Using a description in words to describe a repeating pattern.
- Using ratio notation to describe a repeating pattern.
- Using a ratio table to scale up quantities.
- Drawing and using a rectangular bar model to portion out cost and other quantities.
- Using two ratio tables simultaneously to make comparisons.
- Using ratio tables to develop a version of the unitary method.
- Developing shortcuts using one-step multiplication and division and a calculator if appropriate.

Time is spent working between the contexts and their associated models. Students will also be asked questions that force them to go back from their mathematical models and workings to the context. Often it is the context that allows students to decide whether or not the maths they are doing is allowed/makes sense.

The usual shortcuts associated with working on ratios, such as simplify/cancel down/unitary method, are deliberately not mentioned overtly, as often students do not know how or why such methods work. Through working on this section students are likely to evolve their own shortcuts (which may look like those mentioned) but these will have some basis in common sense on the part of the learner.

Sub sections

<table>
<thead>
<tr>
<th>Questions</th>
<th>Intended emphasis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1–4 Workbook exercises 3.1, 3.2</td>
<td>i) Describing repeating bead patterns by: a) describing in words, b) drawing their own designs and c) using numbers. The focus is on working across these 3 representations from one to the other. ii) The idea that a repeating bead pattern has a number of ratios associated with it (i.e. 3 : 2, 6 : 4, 9 : 6) is introduced.</td>
</tr>
<tr>
<td>Q5–7 Workbook exercises 3.3, 3.4</td>
<td>i) Working with two strategies (models) for scaling ratios. Carol’s strategy involves writing the numbers going across the page, similar to looking at a necklace going across the page. Don’s strategy involves presenting the numbers vertically in a ratio table. ii) With both methods students consider what multiple ‘lots of’ these numbers will be, and what they would look like in the context of the necklace.</td>
</tr>
<tr>
<td>Q8–11 Workbook exercises 3.5, 3.6</td>
<td>i) Working with the rectangular bar model (a rectangular pizza). Through sharing out pizza and sharing out the cost according to the amount eaten, students draw and portion up rectangular bars. The cost becomes the length of the bar. ii) Working on refining trial and improvement strategies for dividing out the cost. iii) The cost can be represented by the length of the bar.</td>
</tr>
<tr>
<td>Q12–22</td>
<td>i) The ratio table model is further developed, so that students experience using two ratio tables simultaneously as a way of comparing the top speeds of various animals. This involves students making decisions about ‘helpful’ numbers to aim for when wanting to make comparisons. ii) Developing a ratio table version of the ‘unitary’ method when converting speeds to metres per second and miles per hour. This enables the top speeds of the human and the cheetah to be put into context.</td>
</tr>
<tr>
<td>Q23–27</td>
<td>i) The context of exchange rates and comparing the relative prices of goods in the UK, the US and Europe is used. This continues the need to work with decimal amounts, as touched on in the previous section. ii) Some students may continue to use the ratio table, now using a calculator to work out some of the interim additions and multiples. Other students may develop shortcuts involving one-step multiplications and divisions.</td>
</tr>
</tbody>
</table>

Resources

Workbook exercises 3.1 to 3.6 in Workbook
Coloured beads/tiles
Internet access to research animal speeds and price comparisons
Mini-whiteboards
Lesson 1: What’s in a necklace? The jewellery company

Possible lesson outline

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td></td>
<td></td>
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<tr>
<td>Q2</td>
<td></td>
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<tr>
<td></td>
<td>Sharing students’ descriptions of how they see the patterns.</td>
<td>It will be helpful if students come to the board to show where they are seeing their pattern.</td>
</tr>
<tr>
<td></td>
<td>Students individually consider Q2 parts a) and b) before reviewing as a class.</td>
<td>Sam’s pattern, although it can be seen, would in fact make a very different necklace. There will be several different strategies around – see Solutions and strategies on pages 40 and 41.</td>
</tr>
<tr>
<td>Individual/Pairs</td>
<td></td>
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</tr>
<tr>
<td>Workbook exercise 3.1 – Q1</td>
<td>Students describe the bead patterns in words, and then using Scarlett’s shorthand.</td>
<td>Look out for students who have seen different patterns, e.g. pink, red, pink versus red, pink, pink. Or 2 pink : 1 red versus 4 pink : 2 red. These are worth bringing to the board.</td>
</tr>
<tr>
<td>Whole class review</td>
<td></td>
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</tr>
<tr>
<td>Workbook exercise 3.1 – Q1</td>
<td>Students demonstrate their ways of recording the patterns for Workbook exercise 3.1 Q1 and decide whether they are the same or different.</td>
<td>Students may start to recognise that some descriptions enable you to re-create the necklace exactly, whereas sometimes the order of the beads is lost. There will be several different strategies around (see Solutions and strategies on page 41).</td>
</tr>
<tr>
<td>Individual/Whole class</td>
<td></td>
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</tr>
<tr>
<td>Workbook exercise 3.1 – Q2</td>
<td>Students individually decide on the amount of beads needed for Workbook exercise 3.1 Q2 before sharing as a class.</td>
<td></td>
</tr>
<tr>
<td>Whole class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discussion focusing around how you might go about setting up a small business as a way of engaging with the context. Students demonstrate at the board where they can see the ratios for Q4. Students draw a third design for 3 : 2 and compare.</td>
<td>Students may have some first- and second-hand experience of setting up small businesses – see Solutions and strategies on page 42. Mini-whiteboards are helpful for Q4. The thought process used leads you to believe you have created a new pattern, but in fact it turns out to be the same as Design 1 or 2.</td>
</tr>
<tr>
<td>Individual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workbook exercise 3.2</td>
<td>Students create designs for the given ratios of beads. Students start to compare their patterns in the search for other different ones.</td>
<td>Some students struggle with the idea that the necklaces are created in a loop, so even if they start differently, they may end up looking exactly the same once made into a loop.</td>
</tr>
<tr>
<td>Pairs</td>
<td></td>
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<tr>
<td>Homework from Workbook exercise 3.2</td>
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</tbody>
</table>
What’s in a necklace?: Solutions and strategies

In this section, there is a deliberate avoidance of trying to describe what the word ratio means. Instead the language associated with it is gradually dropped in, both through the text and through the descriptions that students will supply themselves.

1 a) Students will see and describe their patterns in various ways. They may need to describe and point to the picture at the same time, so others can follow their thinking:

- It goes: black, red, brown, brown; black, red, brown, brown, brown.
- It goes: black, red, three brown.
- It goes: red, then three brown, then black; red, then three brown, then black.
- It’s got two black beads with a red and three browns in between.

Some students may notice what appear to be tiny brown beads in between the larger beads. It’s hard to tell whether these are beads or part of the thread. These can also be recorded if that is the consensus.

It is worth having a record of all the patterns described and starting to consider the precision of these patterns. “Which of these descriptions would help us make the necklace again if it snapped?” can be a helpful question here.

1 b) i) As students start to show Gwen’s thinking in the picture, this will lead to a sectioning up of the necklace. The ‘groups of’ model for ratio is an important feature of this section. It is possible to see Sam’s thinking in the picture, but this will involve going into three of Gwen’s groups. Scarlett’s thinking comes from looking at the whole picture.

The question: ‘Which of these descriptions would help us make the necklace again if it snapped?’ can be used again here. Interestingly, the standard ratio-type description (which Gwen, Sam and Scarlett make use of) is not so helpful as it does not take account of the precise order of the pattern of the beads.

ii) Students may think all the descriptions are OK as they can see each one in the picture. Students may be critical because the descriptions don’t really tell you about the order of the beads. It is important to establish that Sam’s is not really correct, as it double-counts some of the black beads.

b) i) Find in the picture where you can see each of their ideas.

ii) Decide whether you think their patterns can be correct or not for this necklace.

2 You can’t see the entire necklace in this picture, but try to predict the following:

a) If the necklace contains 15 black beads altogether, how many red and how many brown beads do you think it will have?

b) If the necklace contains 54 brown beads altogether, how many black and how many red do you think it will have?

Turn to your workbook and do Workbook exercise 3.1 on pages 19–20.
2 a) Strategies will vary:

Some students may count or draw on from the 10 black beads in the picture as shown:

15 black 15 red 45 brown

Others may realise that they will need the same amount of red beads (= 15) and 15 lots of three brown beads (= 45).

b) Some students may continue with their drawing/counting on strategies:

= 18 black, 18 red

Others may recognise that 54 brown beads = 18 lots of 3, so 18 black and 18 red beads.

Workbook exercise 3.1

1 a) Again, various descriptions will be offered. Below, just one example is given for each necklace:

- Pink, red, pink / pink, red, pink /
- 1 yellow, then 3 green / 1 yellow, then 3 green /
- Bright pink, orange, orange, white, 2 pink / BP, o, o, w, p /
- Spotty, blue, blue, yellow, orange, orange / spotty, bl, bl, y, o /

As demonstrated above, in the interests of economy students are likely to naturally evolve their own shorthand through the process.

b) Again, answers will vary depending on what students see in the picture.

Students may need reminding about what Scarlett’s description looked like. Below is offered the ‘primary’ ratio and one alternative:

2 pink : 1 red
12 pink : 6 red (this is from counting all the beads in this picture)

Note that the technique of counting all the beads in the picture will not always give a ratio consistent with the repeating pattern idea. This should be discussed as it arises. It could be argued that both versions are correct, given that you cannot see the entire necklace. The teacher needs to encourage the idea that all the necklaces are based on repeating patterns, and therefore although it can’t be seen we will assume this is the case.

You may also wish to look out for or drop in the alternative (more formal) representation of a ratio:

Pink : red
2 : 1

This does not need to be forced here, as it is introduced later in the chapter. However, it could be suggested in the interests of clarity, given that it is hard to pick out some of the numbers in the more cluttered versions below:

3 green : 1 yellow 6 green : 2 yellow and so on.
1 BP : 2 O : 1 Wh : 2 P 5 BP : 10 O : 5 Wh : 10 P and so on.
1 : 2 : 1 : 2 2 : 4 : 2 : 4 and so on. (Colours omitted now.)

c) Students’ own guide lines drawn on the pictures, depending on how they see the ratios.

2 a) Students may need to draw a continuation of the necklace up to 12 purple beads.

They may see from the picture/ratio that there are 2 bright blues for every 1 purple and 1 dark green for every 1 purple.

So with 12 purple there will be 24 bright blue and 12 dark green beads.

b) Using methods similar to those in part a):

46 bright blue : 23 purple : 23 dark green.
The jewellery company: Solutions and strategies

3 The purpose of this question is to encourage students to ‘enter the context’ and start to consider what is involved in running a business designing necklaces. Having read the blurb, it may be worth asking questions like: ‘Do any of your family run a small business?’ and ‘How did they get started?’ It is likely that one of the students will have some stories to offer about starting small, turning a room in their house into an office, etc. It may be that some of their friends or family sell stuff on eBay, which could be classed as a small business in some ways. Or they may themselves have been involved in enterprise-type schemes at school. You can ask questions such as: ‘Does it help to do market research?’ etc.

a) Various suggestions, for example:
   - Black and white are popular colours.
   - Good for both men and women. Men wouldn’t want pink.
   - Lots of people wear black, so it will match.

b) Various suggestions, for example:
   - It looks better.
   - It’s easier to make into a pattern.

Carol and Don decided to set up a small business making beaded necklaces and bracelets. They want to target both the male and female markets with their designs. As part of their business plan they have conducted some market research. The research looked into what sorts of beaded necklace designs are popular in terms of the colours used and the length of necklace. One of their first sets of designs is called ‘The Monochrome Set’. This is a series of necklaces made up of only black and white beads. The beads are arranged in repeating patterns. A possible design is shown below:

a) Carol and Don want to target both male and female customers. Suggest some reasons why you think black and white beaded necklaces might be a good seller.

b) Many necklace designers go for repeating patterns in their designs. Suggest some reasons why.
Carol and Don start by sketching designs that use a ratio of three white beads to every two black beads. Here are two partly drawn examples of these designs:

**Design One**

3 : 2

**Design Two**

3 : 2

a) Describe in words any patterns you can see in each of the two designs.
b) Show where you can see the 3 : 2 ratio of white to black beads in Design 1.
c) Show where you can see the 3 : 2 ratio of white to black beads in Design 2.
d) Draw a third design based on a white to black ratio of 3 : 2.
e) Carol thinks that there are only two possible different designs for a white to black ratio of 3 : 2. What do you think?

However, if the necklace is to be continuous then in fact the above designs are exactly the same as Design 2 and Design 1 respectively.

e) After comparing their patterns in part d) with Designs 1 and 2, the students may now be convinced that there are not any new designs for a 3 : 2 ratio.

**Workbook exercise 3.2**

1 a, b & c) The necklaces from question 4 are now presented in a horizontal representation. The purpose of this question is to re-visit the idea that you can produce a variety of designs with the 3 : 2 ratio, but because of the looped nature of a necklace, they may end up looking the same. So it is not possible in part c) to find a 'new' design.

2 Some students really enjoy the challenge of making the designs, but they can experience frustration when designs that they think are different turn out to be the same. There is potential for some good discussions here, student to student.
<table>
<thead>
<tr>
<th>Ratio 3 : 3</th>
<th>Design 1</th>
<th>Ratio 3 : 3</th>
<th>Design 2</th>
<th>Ratio 3 : 3</th>
<th>Design 3</th>
</tr>
</thead>
</table>

Other designs may initially appear different but will end up being the same.

<table>
<thead>
<tr>
<th>Ratio 4 : 2</th>
<th>Design 1</th>
<th>Ratio 4 : 2</th>
<th>Design 2</th>
<th>Ratio 4 : 2</th>
<th>Design 3</th>
</tr>
</thead>
</table>

All others turn out to be the same.

<table>
<thead>
<tr>
<th>Ratio 5 : 2</th>
<th>Design 1</th>
<th>Ratio 5 : 2</th>
<th>Design 2</th>
<th>Ratio 5 : 2</th>
<th>Design 3</th>
</tr>
</thead>
</table>

These two designs appear different but are the same.
Lesson 2: Expanding the business

Possible lesson outline

Objective: To develop strategies for scaling up ratios

Lesson phase | Activity | Comments
---|---|---
Whole class | Introduction | Start with class discussion looking at the new designs. See questions under ‘Comments’ on the right for prompts.
Q5a–c) | | Use questions such as: ‘Which is your favourite?’
Individual/Whole class | Q5d) | ‘Which do you think are suitable for the male market?’
Q6 | Students study Don’s workings and match them with the picture. | ‘Which would cost the most to make?’
Individually they answer Q5d) and then compare answers as a class. Students study Don’s workings and match them with the picture.

Individual | Workbook exercise 3.3 | Students work through Workbook exercise 3.3, employing a variety of strategies. | Students find it difficult to know where to aim for with their figures.

Whole class review | Workbook exercise 3.3 | Students demonstrate and listen to each other’s approaches to Workbook exercise 3.3. | Teachers have found the question ‘What else do you know?’ helps to free students up. After working on this the students often find it easier to answer the set questions.
Homework | Some parts could be set as homework.
Expanding the business: Solutions and strategies

5 a) The repeating pattern is difficult to see. The description will be different depending upon which bead is viewed as the ‘start’.
Using Carol’s markers, a possible description is: ‘Round, small, oval, 2 small, oval, small’, which then repeats itself.

b) The numbers 1, 4 and 2 can be seen in one group of the repeating pattern.
The second line of numbers (2, 8 and 4) can be seen by counting two groups of the repeating pattern, and so on. Students may say ‘It’s double’ just from looking at the numbers; it is important that they also see this in the picture. You cannot see 20, 80 and 40 in the picture, and instead would need to imagine 20 lots of the initial bead pattern.

c) Some students will be able to follow Carol’s thinking:
She doubles it to get 2 lots, then does 20 lots, then adds one more lot on to get 21 lots.
Others may need to work with the initial figures themselves and then compare their working with Carol’s.
Students may not initially recognise the significance of long necklaces being around 150 beads long as the way of knowing when to stop, and hence the need for Carol’s final column showing the total number of beads in each string.

Expanding the business

Carol and Don’s business does well in its first year. They decide to extend the range of designs on offer. They also start to make the necklaces in two sizes, short and long.
Here are some samples from their new range of designs.

Design A – Long
Design B – Long
Design C – Long
Design D – Short
Design E – Short
Design F – Short

The price of different types of bead can vary. Carol wants to estimate the cost of beads for each necklace in the designs shown. In order to do this she needs to work out how many of each type of bead is used to make each necklace.

She works on the basis that long necklaces usually contain around 150 beads in total, whereas short necklaces usually contain around 80 beads in total.

Here are her workings out for Design A.

a) Describe in words any patterns you can see in the beads.
b) Describe where you can see the numbers she has recorded within the picture.
c) Explain how you think she has come up with her final figures of 21, 84 and 42.
d) Short necklaces are usually made up of around 80 beads in total. Use Carol’s method to work out how many of each type of bead would be needed for the short version of Design A.
The price of different types of bead can vary. Carol wants to estimate the cost of beads for each necklace in the designs shown. In order to do this she needs to work out how many of each type of bead is used to make each necklace.

She works on the basis that long necklaces usually contain around 150 beads in total, whereas short necklaces usually contain around 80 beads in total.

Here are her workings out for Design A.

\[\begin{array}{cccc}
1 & 4 & 2 & 7 \\
2 & 8 & 4 & 14 \\
20 & 80 & 40 & 140 \\
21 & 84 & 42 & 147 \\
\end{array}\]

**Design A – Long, around 150 beads**

- **a)** Describe in words any patterns you can see in the beads.
- **b)** Describe where you can see the numbers she has recorded within the picture.
- **c)** Explain how you think she has come up with her final figures of 21, 84 and 42.
- **d)** Short necklaces are usually made up of around 80 beads in total. Use Carol’s method to work out how many of each type of bead would be needed for the short version of Design A.

\[\begin{array}{cccc}
1 & 4 & 2 & 7 \\
2 & 8 & 4 & 14 \\
4 & 16 & 8 & 28 \\
8 & 32 & 16 & 56 \\
10 & 40 & 20 & 70 \\
11 & 44 & 22 & 77 \text{ close to 80} \\
\end{array}\]

Some students find it difficult to scale up from 1 group to 10 groups in one step. In the working above, this has been done by doubling as far as 8 groups and then adding on 2 groups.

Note that Carol is writing her numbers across the page, which is a close representation of the horizontal presentation of the line of beads. This horizontal model is also seen later in this section as the rectangular bar model used to proportion out the price of pizzas.

It is also possible to use a ratio table. This strategy is demonstrated in question 6.
6 a) It would be worth students demonstrating at the board the various patterns they can see. This will naturally lead to sectioning up the necklace.
   The rectangle and oval ‘beads’ are easy to see. It is more difficult to pick out the pairs of links joining the rectangle to the oval.
   One possible way of describing the pattern is: ‘Rectangle, 2 links, oval, 2 links / rectangle, 2 links, oval, 2 links / rectangle… and so on.

b) A possible description is:
   1 stands for the rectangle
   4 stands for the links
   1 stands for the oval
   6 stands for the total number of pieces within one group

c) Students will describe this in various ways. One possible description is:
   He did 10 lots. He doubled the amount. He added 1 and 2 to get 3 lots… now he needs to add the 10 lots and the 3 lots.
   Many students will describe what has happened numerically. You can force them to use the context to check that these numerical calculations are indeed valid. The question: ‘What would 10 lots of the pattern look like?’ is helpful in this respect.
   Some students find it difficult to scale up from 1 group to 10 groups in one calculation. They may find it helpful to start with Don’s initial column of 1, 4, 1, 6, and then you can ask them to fill in what else they know. This often leads to the route of doubling to get 2 groups, then 4 groups, then 8 groups, and then adding 2 groups to get 10 groups.

d) Don’s ratio table showing the final column:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>10</th>
<th>2</th>
<th>3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Links</td>
<td>4</td>
<td>40</td>
<td>8</td>
<td>12</td>
<td>52</td>
</tr>
<tr>
<td>Oval</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>60</td>
<td>12</td>
<td>18</td>
<td>78</td>
</tr>
</tbody>
</table>

Don is working out how many beads he will need for a short necklace using Design B.

He prefers to use a ratio table.

Here are his workings for Design B:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>10</th>
<th>2</th>
<th>3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Links</td>
<td>4</td>
<td>40</td>
<td>8</td>
<td>12</td>
<td>52</td>
</tr>
<tr>
<td>Oval</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>60</td>
<td>12</td>
<td>18</td>
<td>78</td>
</tr>
</tbody>
</table>

a) Describe in words any patterns you can see in this necklace.
b) Describe where you can see the numbers 1, 4, 1 and 6 within the picture.
c) How has Don made the other columns in his ratio table?
d) Don wanted to find out how many beads he would need for a short version of Design B. Copy and complete his ratio table.

Turn to pages 23–24 in your workbook and do Workbook exercise 3.3.
Workbook exercise 3.3

1 a & b) It may now be enough for these descriptions to occur verbally, whilst the students are sectioning up the necklaces. Possible descriptions are:
- Design B – 1 rectangle, 1 oval, 4 links
- Design C – 1 black, 1 red, 3 brown
- Design D – 1 round, 12 small, 2 blue, 1 white (worth reviewing, as repeating pattern is harder to spot)
- Design E – 1 blue, 2 small, 1 spotty
- Design F – 2 pink, 1 red
- Design G – 1 yellow, 3 green
- Design H – 1 spotty, 2 blue, 1 yellow, 2 orange

c) Students may naturally opt to use a strategy similar to Carol’s or a ratio table. They may need a push towards one or the other.

Once students have recorded the first line of the ratio, i.e. 1 : 1 : 3 in the case of Design C, they may claim to be stuck. A question that teachers have found really helpful at this stage is to ask: ‘What else do you know?’ This has the effect of freeing up the students. They can then be prompted to address the question of what total they are aiming at.

The solutions below are shown in a ratio table, but any reasonable strategy is still acceptable.

Design B:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Links</td>
<td>4</td>
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<td>80</td>
<td>20</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>Oval</td>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>60</td>
<td>120</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Another strategy students may use is to work out how many ‘groups’ of the base ratio are needed and then work within the base ratio. For example, in the case above there are 4 links for every 1 rectangle, so for 25 rectangles there will be 4 lots of 25 = 100. And there is 1 oval for every 1 rectangle, so 25 ovals with 25 rectangles.

Design C:

<p>| | | | | |</p>
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Black</td>
<td>1</td>
<td>2</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
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<tr>
<td>Red</td>
<td>1</td>
<td>2</td>
<td>20</td>
<td>10</td>
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<tr>
<td></td>
<td>30</td>
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<td></td>
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</tr>
<tr>
<td>Brown</td>
<td>3</td>
<td>6</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td></td>
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</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>10</td>
<td>100</td>
<td>50</td>
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<td></td>
<td>150</td>
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Design D:

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Large</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td></td>
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</tbody>
</table>

Obviously working in this way, doubling and adding columns, is not efficient, but for some students this is what they feel secure with. Others may want to develop shortcuts. They may see the multiplicative possibilities and want to use a calculator.

The remaining solutions are shown using one-step calculations:

Design E blue : small : spotty : total

<p>| | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>(× 20)</td>
<td>20</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Design F pink : red : total

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(× 26)</td>
<td>52</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

Alternatively they may make this necklace 81 beads long in total. (i.e. 54 pink, 27 red, 81 in total.)

Design G yellow : green : total

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(× 20)</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

Design H spotty : blue : yellow : orange : total

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(× 25)</td>
<td>25</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>
Lesson 3: Unthreaded necklaces. Pablo’s pizza parlour

Possible lesson outline

Objective: To begin to develop strategies for scaling down ratios
To explore ways of sharing out the cost of pizzas

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class</td>
<td>Review of homework if appropriate. Students individually come up with and test possible combinations to see if they work. Students describe and listen to each other’s approaches.</td>
<td>Students may need beads (counters) to enable them to visualise and then experiment with possible combinations. Trial and improvement can prove quite frustrating. Look out for more sophisticated but understandable strategies that can be shared with the whole class.</td>
</tr>
<tr>
<td>Introduction Q7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Individual</td>
<td>Students work through Workbook exercise 3.4. Their ‘journeys’ to a possible repeating pattern will vary.</td>
<td>The ratio table may prove a helpful and popular strategy.</td>
</tr>
<tr>
<td>Workbook exercise 3.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole class review</td>
<td>Students demonstrate their own and listen to each other’s approaches to Workbook exercise 3.4.</td>
<td></td>
</tr>
<tr>
<td>Whole class</td>
<td>Students demonstrate their ways of cutting up the circular pizza into 8 slices. Students demonstrate how to work out how much each person will pay. Students reveal various ways of cutting into 9 pieces and of finding the cost.</td>
<td>Look out for students who cut across the pizza into strips that are not equal in size. Several students will favour the natural strategy of halving, halving and halving again. Mini-whiteboards are helpful here. Look out for trial and improvement when splitting the cost into 9 – see Solutions and Strategies on page 54.</td>
</tr>
<tr>
<td>Introduction Q8 Q9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematical focus
Using the ratio table and other strategies to scale down a ratio. Using trial and improvement and developing other strategies to share out the cost of a pizza.

a) See if you can come up with a possible repeating pattern for the beads in bag F.
b) See if you can come up with a possible repeating pattern for the beads in bag G.

Turn to pages 25–27 in your workbook and do Workbook exercise 3.4.
Unthreaded necklaces

Two new necklaces from Don and Carol's children's range arrive in their bags unthreaded. These are shown below:

Carol carefully counts the number of each type of bead within each bag.
Bag F contains 30 pink beads and 20 green beads.
Bag G contains 8 blue beads, 12 white beads and 16 lilac beads.

Carol spreads the beads out and tries to work out what each necklace could look like. She knows each one will be based on a repeating pattern.

a) See if you can come up with a possible repeating pattern for the beads in Bag F.
b) See if you can come up with a possible repeating pattern for the beads in Bag G.

Turn to pages 25–27 in your workbook and do Workbook exercise 3.4.

Chapter 3: Ratio

Unthreaded necklaces: Solutions and strategies

7 a) Some students will want to use the photograph or even actual beads/counters as a way of grouping the beads together. They may start with trial and error, i.e. 2 pink beads with 1 green bead… this grouping leaves 5 green beads left over. Other students may instinctively try 3 pink with 2 green, because of the numbers 30 and 20. Now the grouping will work.

An alternative to offer is Carol's method or the ratio table method but in reverse. Setting up the initial line and asking the question: 'What else do I know?' can help.

Carol's method:

Pink : green
30 : 20
15 : 10  (each of these amounts can be made into 5 groups)
3 : 2

Using a ratio table:

<table>
<thead>
<tr>
<th>Pink</th>
<th>30</th>
<th>15</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>20</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

Many students will happily reduce 30 : 20 to 15 : 10, but then find it difficult when halving does not work. Some trial and error to find 3 : 2 may be necessary. Other students will happily go from 30 : 20 to 3 : 2 in one step.

b) Again, several strategies are possible: trial and error (using actual beads/counters or the picture to investigate possible groupings); Carol's method; or a ratio table.

Using a ratio table:

<table>
<thead>
<tr>
<th>Blue</th>
<th>8</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>12</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Lilac</td>
<td>16</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

In this case halving and halving again gives the simplest version, so students will naturally find this easier.
Workbook exercise 3.4

Answers only are given below. Students will be employing a variety of strategies (see question 7 answers on page 51 for examples.)

1 a) 12 o : 30 g 6 : 15 2 : 5
   b) 2 orange to 5 green, and these could be arranged in three different ways (see Workbook exercise 3.2 for possible designs with a 5 : 2 ratio).

2 a) Bag A – 10 p : 20 s 1 : 2 1 pink to 2 silver
   (only one possible pattern if you draw it)
   b) Bag B – 15 b : 25 g 3 : 5 3 brown to 5 gold
   c) Bag C – 12 w : 24 g : 20 b 6 : 12 : 10 3 : 6 : 5
   3 white to 6 green to 5 blue
   d) Bag D – 24 s : 84 b 2 : 7 2 silver to 7 black
   e) Bag E – 18 s : 27 b : 45 g 2 : 3 : 5 2 silver to 3 black to 5 gold

The answers in questions in question 2 show a base ratio for each design; students may want to draw a possible arrangement of the beads.

Pablo's pizza parlour

Kate and Pam have been friends since primary school. They both work in town and regularly meet up for a lunchtime pizza at one of Pablo's restaurants. The lunchtime menu offers large pizzas, ideal for sharing.

One lunchtime they order the ham and pineapple pizza shown above and request that it is cut into eight slices. Kate eats five of the slices, leaving three slices for Pam.

When they get the bill, Kate and Pam like to make sure that each of them pays for what they have eaten.

a) Draw a picture to show how to share out the pizza.

b) Work out how much each person should pay.
Pablo’s pizza parlour

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One lunchtime they order the ham and pineapple pizza shown above and request that it is cut into eight slices. Kate eats five of the slices, leaving three slices for Pam.

When they get the bill, Kate and Pam like to make sure that each of them pays for what they have eaten.

a) Draw a picture to show how to share out the pizza.

b) Work out how much each person should pay.

Pablo’s pizza parlour: Solutions and strategies

a) Since the total number of slices required is eight, students are most likely to keeping halving the pizza to achieve eight slices:

b) The halving and halving again strategy can also be used to find the price:

Kate pays £4.25 and Pam pays £2.55.
9 a) Pictures will vary:

b) For many students dividing by 9 would prove difficult. The favoured method is to trial and improve and the picture can help with this:

If they split the picture into 3 first, they may also split £7.20 into 3 = £2.40, and then trial and improve to get 1 slice costing 80p.
So Kate should pay £4.00 and Pam should pay £3.20.

9 Kate and Pam decide it is much easier to cut up a rectangular pizza than a round one. So the next week they order the rectangular cheese and tomato pizza shown here. This pizza costs £7.20.
Kate thinks she will want about five pieces, while Pam only wants four. So they decide to cut the pizza into nine equal slices.

a) Draw a picture to show how to share out the pizza.
b) Share the cost of the pizza between Kate and Pam in the ratio 5 : 4.
# Lesson 4: Pablo’s pizza parlour. Using the bar model for ratio problems

## Possible lesson outline

**Objective:** To refine and practise strategies for sharing out the cost of a pizza.

To apply the bar model strategy to a range of problems.

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class/Pairs</td>
<td>Students discuss in pairs which pizza to choose and how to share it out.</td>
<td>It is worth embracing the context here with questions like: ‘Who goes to restaurants/cafes with their friends?’ ‘Do you share food?’ ‘How would you divide up the bill?’ An interesting question is: ‘What do you think the dots mean underneath the rectangular block?’</td>
</tr>
<tr>
<td>Introduction</td>
<td>Students offer various explanations of Kate’s jottings.</td>
<td></td>
</tr>
<tr>
<td>Q10</td>
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</tr>
<tr>
<td>Individual</td>
<td>Students practise drawing the pizzas and splitting them into different numbers of parts. They use a variety of strategies to then work out the cost. Plain paper is important here as it forces students to start with the whole and divide it up, as opposed to just drawing the number of parts they need.</td>
<td></td>
</tr>
<tr>
<td>Workbook exercise 3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole class review</td>
<td>Students demonstrate their own and listen to each other’s approaches to Workbook exercise 3.5. Encourage students to try and take on board a slightly more efficient way of trialling and improving, where possible, as long as it makes sense to them.</td>
<td></td>
</tr>
<tr>
<td>Workbook exercise 3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole class Q11</td>
<td>Discussion of the order in which the student in Q11 made her calculations. Students compare this with Kate’s serviette. Students individually draw a bar and use it to share out the sweets, and then show their methods. Hear some ideas about how they intend to approach Workbook exercise 3.6. The questions ‘How are they the same?’ and ‘How are they different?’ can help students to refine their comparisons. Mini-whiteboards are useful here.</td>
<td></td>
</tr>
<tr>
<td>Workbook exercise 3.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Individual</td>
<td>Students individually practise using the bar model to solve various ratio problems.</td>
<td></td>
</tr>
<tr>
<td>Workbook exercise 3.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paired review</td>
<td>Students describe their strategies and explain their thinking to their neighbour. Some students may have found it difficult when it comes to, say, 12 pieces. Halving, halving again and then trial and improvement for dividing by 3 may help.</td>
<td></td>
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</tbody>
</table>

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**Mathematical focus**

Working with the bar model to divide quantities in a given ratio.

---

Kate and Pam decide it is much easier to cut up a rectangular pizza than a round one. So the next week they order the rectangular cheese and tomato pizza shown there. This pizza costs £7.20.

Kate thinks she will want about five pieces, while Pam only wants four. So they decide to cut the pizza into nine equal slices.

**a)** Draw a picture to show how to share out the pizza.

**b)** Share the cost of the pizza between Kate and Pam in the ratio 5:4.
Chapter 3: Ratio

10 a & b) These two questions are designed to encourage students to ‘enter the context’ and visualise the situation of sharing out pizzas in this way. Some students may go as far as drawing and cutting up their pizza. Some may only want to split their pizza in half and will need encouragement to believe that you might want to do otherwise. Some may have experience of sharing out pizzas and other kinds of food in this way at restaurants. The important idea here is that the context is imaginable, even if not strictly what they would do.

c) Descriptions will vary, for example:
   - She split it into 5 equal pieces. She imagined each piece cost £2, but this makes £10.
     She knocked 10p off each piece. This made £9.50. She must have known this was the price, because she has done 2 lots of £1.90 to get £3.80 next to Kate, and 3 lots of £1.90 which is £5.70 next to Pam.
   - It cost £9.50 so they must have had the Pepperoni pizza.

Workbook exercise 3.5

1 a) Some students will draw the whole rectangle first, then put in the divisions. This models the strategy they may then use to work out the cost of one slice. These students are more likely to draw the same-sized whole pizza for each part, which would be the case in the restaurant. This does not need to be insisted upon, but is something to be aware of.

Some students draw one slice, then scale up to get the whole pizza. They may end up with different-sized wholes when you look across the pizzas drawn for each part. This does not particularly matter on this occasion, but can be misleading in general.

Workbook exercise 3.5

1 a) If you and the person next to you were to share one of these pizzas, decide between you which one you would choose.
   b) How would you cut it up?
   c) When Kate and her friends go to this restaurant, it is always Kate who works out how to split the bill. Below you can see her calculations on a serviette:

Describe what these calculations tell you about the trip to the restaurant.

   d) Which pizza did they have?

Turn to pages 28–29 in your workbook and do Workbook exercise 3.5.
b) Again, once the picture is drawn several students will adopt a trial and improvement strategy, likely to involve halving where possible, to find the cost of one piece. Some may be able to do the division in one go. The pictures below give some indication of these strategies:

Some students may point out that you would not bother working out to the nearest penny, particularly if you were going to give a tip. This is great as it shows that they are still thinking in the context.

Answers only follow for the remaining dates:
13 Feb: Kate should pay around £1.90; Pam should pay around £5.70.
1 Mar: Kate should pay around £2.55; Pam around £3.40; Lesley around £4.25.
16 Mar: Kate should pay £3.60; Pam should pay £3.60.
3 Apr: Kate should pay around £2.40; Pam around £3.60; Lesley around £4.80.
Using the bar model for ratio problems: Solutions and strategies

11 a) Descriptions will vary. The likely order is: (i) draw the bar; (ii) put £0 and £180 at the start and the end of it; (iii) write down the numbers 10 to 90 (their first guess, which is too small) and mark it with a cross; (iv) write down the numbers 20 to 180; (v) work out that 5 pieces makes £100 and 4 pieces makes £80.

b) Various comparisons can be drawn.

c) 120 cm: 75 cm

Using the bar model for ratio problems

A teacher was marking test papers for her Year 10 class. One of the questions was:

Share £180 in the ratio 5 : 4

A number of the class had left this question out.

Below you see the strategy used by one of the students:

a) Describe the order in which this student may have written things down.

b) Compare this strategy with the method Kate used on her serviette in question 10.

c) In both cases a rectangular bar has been drawn. This is known as the ‘bar model’. Use a similar method by drawing a rectangular bar to share out a giant, 120 cm length, block of chocolate in the ratio 3 : 5.

Turn to pages 30–32 in your workbook and do Workbook exercise 3.6.
**Workbook exercise 3.6**

The students are asked to draw a rectangular bar to help them answer the question. This is worth all students engaging with, but does not need to be a sustained method if they have other valid preferences. (Some may still be using a ratio table.) These bars are drawn with the end of the blocks labelled, like a number line. Some students may prefer to label inside the blocks.

1 & 2

![Image](image1.png)

Answers only for the remaining questions:

1. \[2 : 3 : 7 = 12 \text{ parts} \]
   \[\text{Pupils raise } £490\]

2. \[\text{Answers only for the remaining questions:}\]
   \[\begin{align*}
   3 & : 3 : 7 = 12 \text{ parts} \quad £840 \div 12 = £70 \text{ each} \quad £140 : £210 : £490 \\
   \text{Pupils raise } & £490 \\
   4 & a) \quad 65\% \\
   & b) \quad 35 : 65 \quad 7 : 13 \\
   & c) \quad 7 \, b : 13 \, w \quad 14 \, b : 26 \, w \quad 28 \, b : 52 \, w \quad 52 \, \text{white beads} \\
   5 & 1 \, B : 6 \, L = 7 \, \text{parts} \\
   \quad 100 \, \text{ml} \, B : 600 \, \text{ml} \, L = 700 \, \text{ml} \\
   \quad 500 \, \text{ml} : 3000 \, \text{ml} = 3500 \, \text{ml} \\
   \quad 500 \, \text{ml of blackcurrant juice with } 3000 \, \text{ml of lemonade} \\
   \end{align*}\]

6. a) Opinions will vary. Some students may have experience of working in salons.

   b) \[4 : 1 \quad £4 : £1 = £5\]
   Some students will find it hard to get their head around sharing £4 in the ratio 4 : 1 (because the number of pounds (4) is smaller than the total number of parts which is 5). The rectangular bar enables them to trial and improve to get 80p for each part.
   Stylist gets £3.20; trainee gets 80p.

   c) Strategies will vary. Some students may share out the extra £2 in the ratio 4 : 1, i.e. halve the previous answers to give £1.60 : £40p, and then add these onto the previous answers to give £4.80 : £1.20.

7. If the students draw their bars the same size (which they should strictly speaking be doing, because the whole is the same in each case) then they should realise quite quickly that there is no need to work out the actual rental cost. The \(\frac{1}{7}\)th block is smaller than the \(\frac{1}{5}\)th block. So, Devonshire Cottages take less of a cut and it is better to choose them.
Lesson 5: Who is the fastest of them all?

Possible lesson outline

**Objective:** To use ratio tables as a way of comparing animal speeds

**Mathematical focus**

Comparing speeds using ratio tables

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>Students individually, without discussion, have to come up with an order of speed for the animals. Students start to compare their orders and give justifications. Students offer a variety of humorous suggestions.</td>
<td>Answers will vary and cause much debate. These questions are designed to embrace the context, so go with it.</td>
</tr>
<tr>
<td>Individual</td>
<td>Students individually start to interrogate the table and spot some of the straightforward comparisons.</td>
<td>Students will naturally start to ‘scale up’ some of the table entries – see Solutions and strategies on page 62.</td>
</tr>
<tr>
<td>Whole class</td>
<td>Students study the two ratio tables and explain Patricia’s thinking. Similarly for Frank’s work. Hear some ideas about how they intend to approach Q17.</td>
<td>The question: ‘How did she/he decide what to aim for?’ in the ratio tables can be thought-provoking here.</td>
</tr>
<tr>
<td>Individual</td>
<td>Students practise creating and using ratio tables to compare speeds.</td>
<td>Creating the table and knowing where to aim are difficult. The question: ‘What else do I know?’ can be a helpful starting place for students.</td>
</tr>
<tr>
<td>Whole class review</td>
<td>Students share some of the animal comparisons they made. Students brainstorm and work on Q18 as a class, making joint decisions about how to proceed.</td>
<td>Students find it difficult to know what to aim for. Seeing how far both the hare and the tortoise can go in a minute is one way forward.</td>
</tr>
</tbody>
</table>
Who is the fastest of them all?

12 Over the years scientists have measured and recorded the top speeds achieved by various animals.

Look at the list of animals below and try to rank them in order of speed, fastest first.

- Elephant
- Human
- Grizzly bear
- Cheetah
- Greyhound
- Snail
- Lion
- Hare
- Giant tortoise
- Horse

13 a) Give some reasons why it is not possible to line these animals up and race them against each other.

b) Make some suggestions as to how you could measure the speeds of the various animals.

Who is the fastest of them all?: Solutions and strategies

12 Answers will vary. There is often agreement at the extremes (i.e. cheetah and snail) and heated discussion surrounding ranking the lion versus the horse versus the elephant. It is intended for this to be left up in the air at this stage.

13 a) Suggestions will vary, for example:
- The animals will eat each other.
- They won’t all run in a straight line.

Like question 12 this is an ‘entering the context’ device, so go with it.

b) Suggestions will vary, for example:
- Film the animals running and find a way to work out the speed from the film.
- Make a rule that you are going to record the maximum speed for each animal.
- Put marks in the ground and use a stopwatch.
- Attach a speedometer linked to a computer.
14 a) Students start to make comparisons in a large variety of ways using the recordings, for example:

   The lion’s 400 m in 18 s is the same as 200 m in 9 s and 100 m in 4 \(\frac{1}{2}\) s.

   This is not as good as the cheetah, which does 160 m in 5 s.

b) Various informal comparisons, for example:

   The snail’s 12 m in 15 min is the same as 24 m in 30 min, which is much slower than the giant tortoise’s 136 m in 30 min.

15 a) Various descriptions. Students may tend to describe each stage step by step. They may not initially recognise the significance of finding out the times for the horse and the greyhound over the same distance. For example:

   For the horse Patricia doubled it. This tells her how long it would take the horse to run 1000 m.

   Comments about the fact that it may actually take the horse more than double the time are worth discussing, as this shows the students are keeping an eye on the reality of the situation.

   For the greyhound Patricia doubled 400 m, halved 400 m, and added them together.

   Again, it is worth discussing whether this is acceptable, both within the bounds of reality and within the world of mathematics. It is also worth discussing the idea that a lot of maths is about finding a model that is not always a perfect fit in the real world, but that provides a way forward in making comparisons and therefore speculating about real-life events.

b) Students should show some recognition that once the distances are the same, you only need to compare the time. So on this reckoning the horse is slightly faster than the greyhound. Given that both these animals are trained as racers, it would actually be possible to race them against each other. Interesting thought!

The following table shows some recordings made by scientists.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Distance (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elephant</td>
<td>300</td>
<td>27</td>
</tr>
<tr>
<td>Lion</td>
<td>400</td>
<td>18</td>
</tr>
<tr>
<td>Human</td>
<td>22</td>
<td>1.8</td>
</tr>
<tr>
<td>Cheetah</td>
<td>160</td>
<td>5</td>
</tr>
<tr>
<td>Horse</td>
<td>500</td>
<td>25</td>
</tr>
<tr>
<td>Hare</td>
<td>260</td>
<td>12</td>
</tr>
<tr>
<td>Greyhound</td>
<td>400</td>
<td>22(\frac{1}{2})</td>
</tr>
<tr>
<td>Grizzly bear</td>
<td>400</td>
<td>30</td>
</tr>
<tr>
<td>Snail</td>
<td>12</td>
<td>15 min</td>
</tr>
<tr>
<td>Giant tortoise</td>
<td>136</td>
<td>30 min</td>
</tr>
</tbody>
</table>

This table can help you decide who was right when you put the animals in order of speed.

a) Were you right with your 1st and 2nd place animals?
b) Were you right with your last and next to last animals?

Patricia wanted to find out who was faster between the horse and the greyhound. This is what she did:

```
Horse

<table>
<thead>
<tr>
<th>Metres</th>
<th>500</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs</td>
<td>25</td>
<td>50</td>
</tr>
</tbody>
</table>

Greyhound

<table>
<thead>
<tr>
<th>Metres</th>
<th>400</th>
<th>800</th>
<th>200</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs</td>
<td>22(\frac{1}{2})</td>
<td>45</td>
<td>11(\frac{1}{4})</td>
<td>56(\frac{1}{4})</td>
</tr>
</tbody>
</table>
```

a) Describe what Patricia has done.
b) Explain how this helps Patricia to decide which animal is fastest.
16. Frank does the following work to help him compare the horse and the greyhound.

<table>
<thead>
<tr>
<th>Horse</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metres</td>
<td>500</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Secs</td>
<td>25</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Greyhound</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metres</td>
<td>400</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Secs</td>
<td>22 1/2</td>
<td>11 1/2</td>
<td>5 1/8</td>
</tr>
</tbody>
</table>

a) Describe what Frank has done.
b) Explain how this helps Frank to decide which animal is fastest.

17. Choose some of the animals you were not sure about and use ratio tables to compare their speeds.

18. Find a way to compare the hare and the tortoise. How much faster is the hare than the tortoise?

16 a) Various descriptions.

Frank uses 500 m to work out 100 m. This represents a significant shift for some students who are very comfortable with using the ratio table to double, halve and add quantities, but find division (other than by 2) a very difficult idea to grasp. They are more inclined to work out 250 m and 125 m.

For the greyhound Frank halves and halves again; this is much more of a ‘student’ strategy.

b) This helps Frank to get an idea of how long it might take the horse and the greyhound to run 100 m. Again, you can see that on this reckoning the horse would take slightly less time and is therefore faster.

17. Students’ own choices about which animals to compare. They may struggle to see which distance to aim for in order to compare the times. The question: ‘What else do you know?’ can be used once the ratio table has been set up to help decide which distance (or indeed time) to go for.

18. Some students may want to calculate the speed of each animal by using a calculator to divide distance by time. Others may continue with the ratio table idea. However, it is hard to know what to aim for as the common distance or time. Going for a minute could be helpful:

<table>
<thead>
<tr>
<th>Hare</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metres</td>
<td>260</td>
<td>520</td>
<td>1040</td>
</tr>
<tr>
<td>Seconds</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tortoise</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metres</td>
<td>136</td>
<td>45 1/3</td>
<td>4.53</td>
</tr>
<tr>
<td>Minutes</td>
<td>30</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Talking about the difference between 4.53 m marked out on the classroom floor and 1300 m (nearly three times round an athletics track) helps to give some idea about how much quicker the hare is.
# Lesson 6: World record speeds and beyond? Animals on the motorway

**Possible lesson outline**

**Objective:** To consider the top speeds of humans and cheetahs in the context of world records and motorways

### Mathematical focus
Developing the use of the ratio table

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
</tr>
</thead>
</table>
| Whole class Introduction Q13b re-visited Q19 | Students consider how the animal speed data was collected.  
Students describe Frank’s method and discuss how it can be that this speed is faster than the world record. |
| Individual Q20              | Students individually practise converting some of the animal speeds into metres per second. |
| Whole class review Q20      | Students demonstrate their own and listen to each other’s approaches.  
They discuss the interpretation of their results. |
| Whole class Q21             | Students discuss their informal ideas about whether a cheetah could keep up on a motorway (e.g. from wildlife films) and recognise the need to convert the measurements. |
| Individual/Whole class Q21  | Students individually look at Patricia’s method and then discuss her thinking as a class. |
| Individual/Pairs Q22a)      | Students individually or in pairs choose other animals and practice converting speeds into miles per hour.  
Set up homework Q22b)–d). |

- **Comments**
  - It is worth re-visiting Q13b) and discussing how the data was collected. This can be researched on various websites.
  - Students may not like the idea that a mathematical model is not always an exact fit to the reality.
  - Students should be encouraged to continue with the ratio table here, if that is what makes sense. Some students may be able to justify why you can just divide the metres by the seconds and may want to use a calculator. This is also fine.
  - The question: ‘What does that mean if they were to race against each other?’ can be helpful to encourage students to focus on the contextual meaning of their calculations.
  - Questions like: ‘Would a cheetah’s top speed look out of place on a motorway?’ are helpful to generate discussion of the context.
  - Some students will struggle with these conversions particularly with converting km per hour to miles per hour. Drawing 115.2 km as a number line and visualising jumps of 8 km may help.
World record speeds and beyond?

19 The table shows animals recorded over distances in which they achieved their maximum speed. So, for example, the human speed was measured over 22 metres of a 100 m sprint. The lion and the elephant were measured over distances when they were in the act of charging.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Distance (metres)</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elephant</td>
<td>300</td>
<td>27</td>
</tr>
<tr>
<td>Human</td>
<td>22</td>
<td>1.8</td>
</tr>
<tr>
<td>Horse</td>
<td>500</td>
<td>25</td>
</tr>
<tr>
<td>Greyhound</td>
<td>400</td>
<td>30</td>
</tr>
<tr>
<td>Snail</td>
<td>12</td>
<td>15 min</td>
</tr>
<tr>
<td>Lion</td>
<td>400</td>
<td>18</td>
</tr>
<tr>
<td>Cheetah</td>
<td>160</td>
<td>5</td>
</tr>
<tr>
<td>Hare</td>
<td>260</td>
<td>12</td>
</tr>
<tr>
<td>Grizzly bear</td>
<td>400</td>
<td>13 and a bit</td>
</tr>
<tr>
<td>Giant tortoise</td>
<td>136</td>
<td>30 min</td>
</tr>
</tbody>
</table>

Frank wonders how the human speed compares with the world record for running 100 metres.

He starts by converting the human speed into metres per second:

\[
\text{Human speed} = \frac{22 \text{ metres}}{1.8 \text{ seconds}} = 12 \text{ metres per second}
\]

20 a) Draw your own ratio tables to help you answer the following questions:

a) Convert the elephant’s speed into metres per second. Do the same for the grizzly bear. Who would win that race?

Elephant:

<table>
<thead>
<tr>
<th>metres</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>27</td>
</tr>
</tbody>
</table>

Grizzly bear:

<table>
<thead>
<tr>
<th>metres</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>30</td>
</tr>
</tbody>
</table>

b) How much faster is a horse than a greyhound in metres per second?

Horse:

<table>
<thead>
<tr>
<th>metres</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>25</td>
</tr>
</tbody>
</table>

Greyhound:

<table>
<thead>
<tr>
<th>metres</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>22½</td>
</tr>
</tbody>
</table>

So a horse is 3 m/s faster than a greyhound, i.e. covers 3 more metres in every second. That sounds quite a lot faster, when you put it that way.

b) Frank is quite surprised by the results for the human. He knows that the world record for sprinting 100 metres is just under 10 seconds. He thinks this is like doing 10 metres in 1 second. The calculation he did above says a human can run around 12 metres in 1 second.

How can you explain this difference?

b) This discrepancy is due to the fact that the measurements were taken when the ‘animals’ were running at maximum speed. So presumably the human measurement was taken over a small part of the 100 metres, i.e. during the middle of the race when the athlete had reached their maximum speed. In other words, the start of the race (the setting off and getting up to speed part) is not included.

20 a) As with all ratio tables, strategies will vary. Below is just one possible ‘journey’ in each case.

(Informal divisions like \(100 \div 9 = ‘11 \text{ and a bit}’\) are more than acceptable here.)

Elephant:

<table>
<thead>
<tr>
<th>metres</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>27</td>
</tr>
</tbody>
</table>

Grizzly bear:

<table>
<thead>
<tr>
<th>metres</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>30</td>
</tr>
</tbody>
</table>

The grizzly bear will cover around 2 metres more in every second than the elephant. Quite hard to imagine given the size of these animals.

b) Students have already compared a horse and a greyhound in questions 15 and 16, but in those cases they used a different common distance. Now they are being asked to make the time of the comparison consistent.

Horse:

<table>
<thead>
<tr>
<th>metres</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>25</td>
</tr>
</tbody>
</table>

Greyhound:

<table>
<thead>
<tr>
<th>metres</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>22½</td>
</tr>
</tbody>
</table>

So a horse is 3 m/s faster than a greyhound, i.e. covers 3 more metres in every second. That sounds quite a lot faster, when you put it that way.
c) Lion:

<table>
<thead>
<tr>
<th>metres</th>
<th>400</th>
<th>200</th>
<th>22 and a bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>18</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

Hare:

<table>
<thead>
<tr>
<th>metres</th>
<th>260</th>
<th>130</th>
<th>65</th>
<th>21 and a bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>seconds</td>
<td>12</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

These are actually quite close: the lion does around 1 more metre in every second than the hare. Interesting when you think about the relative sizes of these two animals.
Animals on the motorway

21 Patricia wonders how the animal speeds compare with the speed of a car. She starts with the fastest animal, the cheetah, and tries to convert this speed into miles per hour.

<table>
<thead>
<tr>
<th>Metres</th>
<th>160</th>
<th>320</th>
<th>480</th>
<th>115200</th>
<th>1152 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>1 min</td>
<td>1 hr</td>
</tr>
</tbody>
</table>

a) Describe what Patricia has done so far.

b) 5 miles is roughly 8 km. Use this fact to try to convert 115.2 km per hour into miles per hour.

c) Do you think a cheetah’s top speed would look out of place on a motorway?

22 a) Choose three other animals from the table and convert their speeds into miles per hour.

b) How much faster is the hare than the tortoise in miles per hour?

c) What about the human speed in miles per hour?

d) Do you think miles per hour is a good way to look at animal speed or not?

Animals on the motorway: Solutions and strategies

21 a) Descriptions will vary, for example:

She wanted to see how far the cheetah could go in an hour. So she doubled, then timesed by 6 to get up to a minute. She then timesed by 60 to get up to an hour. Then she changed the metres into kilometres by dividing by 1000.

b) You need to divide by 8, presumably on a calculator, then multiply by 5.

115.2 km per hour = 72 miles per hour.

Some students may prefer these calculations to be set out as a ratio table so they can perform interim stages, even when using a calculator.

c) Clearly a cheetah would look out of place on a motorway! However, speed-wise that would be about right.

22 a) Students’ own choices. Ratio tables and calculators may both be in use.

b) Hare: 260 m in 12 s converts to 48.75 miles per hour.

Tortoise: 136 m in 30 min converts to 0.17 miles per hour.

So in miles per hour the hare is about 287 times faster than the tortoise. Presumably the hare would struggle to run for an hour at that pace, and indeed so might the tortoise if the original measurement is also him working at maximum speed.

c) Human: 22 m in 1.8 s converts to 27.5 miles per hour.

Given that the current (as of 1 Sept 2011) world record for the marathon (26 miles) is 2 hours 3 minutes and 59 seconds (Haile Gebrselassie, 2008) for men and 2 hours 15 minutes and 25 seconds for women (Paula Radcliffe, 2003) you can see the discrepancy between the scientific measurement and the reality. This is another example of the conflict between the reality and the mathematical model.

d) Answers will vary. Given that a lot of the animals in the list would struggle to run for an hour and certainly could not sustain their maximum speed for an hour, it does not seem a sensible way to look at it. However, it does provide a way of thinking about animal speeds compared with something we have all experienced: the speed of a car. So in that sense it is a helpful way to look at it.
Lesson 7: Jewellery prices. Buying electrical goods in America. Summary

Possible lesson outline

**Objective:** To compare the prices of items in the UK and abroad

<table>
<thead>
<tr>
<th>Lesson phase</th>
<th>Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class Introduction Q23</td>
<td>Discussion embracing the context of why you might need to charge more when selling goods abroad. Students individually consider what to charge for the necklace before sharing their ideas and methods at the board or in pairs. Students compare their own strategies and prices with Carol’s.</td>
<td>Look out for a range of strategies including the ratio table (see Solutions and strategies on page 69). Encourage students to see one person’s strategy within another’s.</td>
</tr>
<tr>
<td>Individual/Pairs Q24–26</td>
<td>Students individually work out what price to put on the necklace in Q24 and compare their suggestions with their partner’s. Students work out the mark-up from pounds to Euros in Q25. Students work out which label is showing the biggest mark-up from pounds to Euros in Q26.</td>
<td>It may be necessary to compare methods and review opinions on how companies decide on their prices from one country to the next (see Solutions and strategies on page 70). Promote discussion about where the labels are from and the countries of origin of these companies (see Solutions and strategies on page 71).</td>
</tr>
<tr>
<td>Whole class Q27a)</td>
<td>Discussion embracing the context of shopping around on prices before you go abroad. Students suggest (and demonstrate) how John has come to his conclusion.</td>
<td>Ask students to recount examples of where they or their family/friends have done this – see Solutions and strategies on page 72.</td>
</tr>
<tr>
<td>Individual Q27b)</td>
<td>Students work through the table, converting and comparing the US and UK prices.</td>
<td>Look out for two main strategies (see Solutions and strategies on page 72). Students may need visual images to help them recognise the need to divide by 1.59 for conversion from dollars to pounds. Encourage natural use of calculators (for multiplication and division) and mental estimation for subtractions.</td>
</tr>
<tr>
<td>Individual/Pairs Q27c)</td>
<td>Students discuss what they would like to buy. They research the prices in the UK, US and Europe and draw comparisons to see where their chosen products are cheapest to buy.</td>
<td>Best with students working one or two to a machine researching international prices of goods.</td>
</tr>
</tbody>
</table>
Jewellery prices

Carol and Don want to expand their business even further. They start looking into selling some of their necklaces in Europe and in America. They wonder how much to mark up the prices for selling in Europe and in America.

a) Why might they need to charge more for their necklaces in Europe and in America compared with the UK?

b) This necklace retails at £9 in the UK.

Make some suggestions about what price you would put on this necklace:

i) in Europe

ii) in America

(The exchange rates at the time are: £1 = 1.13 Euros; £1 = 1.63 US dollars.)

c) Carol uses a ratio table to help her think about the price. This can be seen below:

<table>
<thead>
<tr>
<th>£</th>
<th>1</th>
<th>10</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro</td>
<td>1.13</td>
<td>11.3</td>
<td>10.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>£</th>
<th>1</th>
<th>10</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>US dollars</td>
<td>1.63</td>
<td>16.3</td>
<td>14.67</td>
</tr>
</tbody>
</table>

Describe what Carol has done.

d) Carol decides to price the necklace at 12 Euros for the European market and 16 dollars for the American market.

How do these prices compare with your suggestions from part b)?

Jewellery prices: Solutions and strategies

23 a) Suggestions will vary:

- You need to take into account the cost of sending them over there.
- It might be a richer country, so you could charge more.

b) i) Students may recognise £1 = 1.13 Euro; £2 = 2.26 Euro... so £9 = 9 lots of 1.13 = 10.17 Euro.

It is important here to keep the ‘lots of’ idea around so that students can make sense of why they are doing what they are doing. They may wish to use a calculator, which is most appropriate here.

Some students may suggest or want to use a ratio table, which indicates that they are starting to realise the power of this strategy for solving a number of seemingly different types of problem. It would be useful to bring this strategy to the board. However, if it does not arise naturally then it is presented in question 23c).

ii) Similarly £1 = $1.63; £2 = $3.26... so £9 = 9 lots of 1.63 = $14.67.

The students will probably need a reminder to then interpret these calculations in terms of what price to charge, i.e. how close would the charge be to these prices? Presumably something would need to be added to take account of transporting the goods, although they do not take up much room. Also, prices are usually very close to a whole number.

So suggested charges may be anything upwards from: i) 11 Euro and ii) $15.

c) Descriptions will vary. Most students will recognise the idea of doing ten lots then taking one lot off.

d) Answers will vary depending on what the students have suggested as the prices to use in part b). This is a good opportunity to explore the context further and for students to bring in their own experiences (e.g. holidays abroad, internet shopping, The Apprentice). For example, Carol is pricing just above the calculated exchange rate prices. Using exchange rates can only be a rough guide because they do fluctuate quite regularly. There are also other considerations like transport costs. The size of the order (from retailers aboard) and the usual cost for such items in those countries also need to be taken into account.

Some students may perceive America to be quite a rich country and therefore you can charge more there. Others may recognise that you can produce these goods quite cheaply in America, so they would need to be competitively priced if any American companies were going to place an order.

Students may bring in their own experiences of buying items on holiday and stories of what seems cheaper or dearer to buy abroad compared with in this country.
24 Students may query which exchange rate to use as a guide to comparing the prices. They could use the one given in the text, or go on the internet to find out the exchange rate for that day.

Answers below are based on the given rate of £1 = 1.13 Euro and £1 = $1.63 and using a ratio table method:

a) | Pounds | 1 | 10 | 20 | 2 | £22 |
   | Euros  | 1.13 | 11.3 | 22.6 | 2.26 | 24.86 Euro |

b) | Pounds | 1 | 10 | 20 | 2 | £22 |
   | US dollars | 1.63 | 16.3 | 32.6 | 3.26 | $35.86 |

Some students may continue with a ratio table method, even though they may start to use a calculator to add some of the numbers. Other students may recognise that they can get there in one move by doing 22 lots (in this case). The role of the teacher here is to expose both methods and question students about whether these are really the same or not.

25 a) Methods of comparing will vary:

Students may recognise informally that £20 : 30 Euro is like £2 : 3 Euro, which is like £1 : 1.50 Euro.

They may want to use a ratio table to scale the first ratio down.

Interpreting this is difficult. You appear to get more Euros for your pound on the jewellery price rate. What does that mean?

Other students may convert a price of £20 into Euros using the rate £1 = 1.13 Euro:

£20 = 20 × 1.13 = 22.6 Euro.

This seems easier to interpret. If you start with the British price (£20), it would cost more for a British person to buy this item in Europe (30 Euro), because the equivalent is really 22.6 Euro.

b) Marked up by 30 – 22.6 = 7.40 Euro.

c) Similar suggestions to question 23a) and d), i.e. cost of transport, standard of living for that export country, competitive pricing, etc.

The necklace shown below retails at £22 in the UK.

a) Suggest what price to charge in Europe.

b) Suggest what price to charge in America.

25 Carol starts looking in other shops to see what the difference is in prices in the UK compared with other countries.

Here is an example of a ticket price in the UK:

a) How do the prices shown on this ticket compare with the exchange rate of £1 = 1.13 Euros?

b) This item was made by a British company and then sold in other countries in Europe. Based on the exchange rate of £1 = 1.13 Euros, this item has been marked up for the European market. By how much has it been marked up?

c) Why might companies mark up their prices when they sell products abroad?
The following price tickets are for a different company:

**26 a)** Again, this is hard to answer without knowing the exchange rate. Students may initially make some informal suggestions. More precisely, using the rate provided earlier of £1 = 1.13 Euro, then this would give:

- £9.99 = 11.29 Euro
- £14.99 = 16.94 Euro
- £19.99 = 22.59 Euro

Using the exchange rate of £1 = 1.13 Euros the items are more expensive in Euros.

Some students may recognise these labels as they come from the shop H&M which is a clothing company which originated in Sweden in 1947. The previous label is for Monsoon, a company that originated in 1973 in London. If you look closely at the label you will see ‘Made in India’ which is where a lot of the clothes on the British high street are made.

**26 b)** The mark-ups are as follows:

- £9.99 ticket: 12.95 – 11.29 = 1.66 Euro
- £14.99 ticket: 19.95 – 16.94 = 3.01 Euro
- £19.99 ticket: 24.95 – 22.59 = 2.36 Euro

This again demonstrates a lack of consistency (other factors having an effect), as the middle-priced item in the UK has the biggest mark-up when you compare with the Euro prices. Students may mention the idea that a lot of clothes prices in the UK and in Europe tend to be close to a whole number, often a multiple of five or ten.
Buying electrical goods in America: Solutions and strategies

27 a) Suggestions will vary, for example:
He knew £100 was like $159, so £131 for the iPod Nano would be way more than $149.
$320 is roughly £200. The iPod Touch (32 GB) costs $299, which is less than $320, but it costs more than £200 in the UK.

b) Students can go either way here. They might decide to change the US price into pounds by dividing by 1.59. They may need a visualisation to be offered, such as jumps of length 1.59 on a number line, so that they can start to develop a sense of why they are dividing. Although this involves division, you are more likely to want to convert in this way because it makes more sense to see the equivalent price in pounds, i.e. your own currency.
Alternatively students may change the UK price into dollars by multiplying by 1.59.
Encourage mental estimation for the amount saved, as that is likely to be what you would actually do: see the equivalent price on the screen of your calculator and from there work out the rough subtraction in your head.

c) Students who have been abroad may have experience of doing rough calculations, or using their phones to do price comparisons. Going to a computer room with some suggested products to research could be quite productive, particularly if they have suggested the products themselves.

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## Buying electrical goods in America

John is going to America on business. He has heard that some electrical goods are cheaper over there than in the UK. John would like a new iPod. He does some research on the internet, looking at the price of iPods in the UK and in America.

<table>
<thead>
<tr>
<th>Product</th>
<th>US retail price</th>
<th>UK retail price</th>
<th>Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPod Nano</td>
<td>$149</td>
<td>£131</td>
<td>Nearly £40 (£37.29)</td>
</tr>
<tr>
<td>iPod Shuffle</td>
<td>$49</td>
<td>£40</td>
<td>Nearly £10 (£9.18)</td>
</tr>
<tr>
<td>iPod Touch – 8 GB</td>
<td>$229</td>
<td>£193</td>
<td>Nearly £50 (£48.97)</td>
</tr>
<tr>
<td>iPod Touch – 32 GB</td>
<td>$299</td>
<td>£254</td>
<td>Nearly £70 (£65.95)</td>
</tr>
<tr>
<td>iPod Touch – 64 GB</td>
<td>$399</td>
<td>£336</td>
<td>Nearly £86 (£85.06)</td>
</tr>
</tbody>
</table>

On the day John looks at these prices, the exchange rate is: £1 = $1.59.

a) John looks at these figures and does a quick calculation in his head. He reckons that it will be cheaper to buy these items in America. What do you think he did?

b) Work out how much he could save on each item if he bought it in America.

c) Use the internet to research the prices of some goods that you are interested in. Compare the prices in the UK, America and Europe to find out where they are cheapest.

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### Summary

**Equivalent ratios**
In this chapter you have looked at various ways of writing ratios. In this necklace you can see a repeating pattern of black, red, brown, brown, brown; black, red, brown, brown, brown.
The ratio of black : red : brown is 1 : 1 : 3.
It is also possible to see a black : red : brown ratio of 2 : 2 : 6.
There are several other ways of seeing the ratio, such as 8 : 8 : 24.
The ratios: 8 : 8 : 24 and 2 : 2 : 6 and 1 : 1 : 3 are all equivalent ratios.
1 : 1 : 3 is the simplified ratio of 8 : 8 : 24.

### Sharing in a given ratio
You learned how to use the rectangular bar model to share out quantities in a particular ratio.
In the problem below, Kate and Pam were sharing the cost of their £9.50 pizza in the ratio 2 : 3.
Kate first drew a rectangular block.
She split it into five parts (two parts and three parts).
She then figured out the cost for each part and worked out a cost for two parts and for three parts.
Summary

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Summary

Through the context of the necklace, students are reminded of how equivalent ratios can be obtained by scaling up and scaling down. The necklace is important as it provides visual imagery of what scaling up and scaling down looks like in terms of being able to see ‘groups of’ the base ratio.

The bar model and the ratio table strategies which were introduced in Chapters 1 and 2 are revisited here. In the Chapter 3 Summary we are reminded how the bar model can be used to share in a given ratio and the ratio table can be used to scale up ratios and as a way of converting speeds. It is hoped that students will be starting to develop a sense of how these two strategies can be used to solve problems in a variety of topic areas. In classroom trials teachers commented on just how useful the ratio table and the bar model were for answering several questions from GCSE Foundation level papers.

Students can be asked to read the Summary and then make some notes where they create their own examples under the headings ‘Equivalent ratios’, ‘Sharing in a given ratio’ and ‘Using a ratio table’.
Using a ratio table

You saw several situations where a ratio table could help you solve a problem. The ratio table below was used to work out how many of each type of bead would be needed in a necklace of 150 beads in total.

<table>
<thead>
<tr>
<th></th>
<th>Rectangle</th>
<th>Links</th>
<th>Oval</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>80</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6</strong></td>
<td><strong>60</strong></td>
<td><strong>120</strong></td>
<td><strong>30</strong></td>
</tr>
</tbody>
</table>

The ratio table below was used to see how far a greyhound doing 400 metres in $22\frac{1}{2}$ seconds would go in 1 second.

<table>
<thead>
<tr>
<th>Metres</th>
<th>400</th>
<th>800</th>
<th>160</th>
<th>17 and a bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seconds</td>
<td>$22\frac{1}{2}$</td>
<td>45</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

A ratio table is a really helpful tool to solve many problems in many topic areas, not just those that obviously mention ratios.