What’s in a necklace?

1. a) Look at the beaded necklace shown above. Describe in words any patterns you can see in the necklace.

b) Here are the responses of some other students to part a):
   
   Gwen said: ‘It’s 1 black for every 1 red and 3 brown.’
   
   Sam said: ‘It’s 3 black to 2 red to 6 brown.’
   
   Scarlett said: ‘I think it’s 10 black : 10 red : 30 brown.’

   i) Find in the picture where you can see each of their ideas.
   
   ii) Decide whether you think their patterns can be correct or not for this necklace.

2. You can’t see the entire necklace in this picture, but try to predict the following:

   a) If the necklace contains 15 black beads altogether, how many red and how many brown beads do you think it will have?
   
   b) If the necklace contains 54 brown beads altogether, how many black and how many red do you think it will have?

Turn to your workbook and do Workbook exercise 3.1 on pages 19–20.
Carol and Don decided to set up a small business making beaded necklaces and bracelets. They want to target both the male and female markets with their designs.

As part of their business plan they have conducted some market research. The research looked into what sorts of beaded necklace designs are popular in terms of the colours used and the length of necklace.

One of their first sets of designs is called ‘The Monochrome Set’. This is a series of necklaces made up of only black and white beads. The beads are arranged in repeating patterns.

A possible design is shown below:

![A possible design of a beaded necklace](image)

**a)** Carol and Don want to target both male and female customers. Suggest some reasons why you think black and white beaded necklaces might be a good seller.

**b)** Many necklace designers go for repeating patterns in their designs. Suggest some reasons why.
Carol and Don start by sketching designs that use a ratio of three white beads to every two black beads. Here are two partly drawn examples of these designs:

a) Describe in words any patterns you can see in each of the two designs.
b) Show where you can see the 3 : 2 ratio of white to black beads in Design 1.
c) Show where you can see the 3 : 2 ratio of white to black beads in Design 2.
d) Draw a third design based on a white to black ratio of 3 : 2.
e) Carol thinks that there are only two possible different designs for a white to black ratio of 3 : 2. What do you think?

Turn to pages 21 and 22 in your workbook and do Workbook exercise 3.2.
Expanding the business

Carol and Don’s business does well in its first year. They decide to extend the range of designs on offer. They also start to make the necklaces in two sizes, short and long.

Here are some samples from their new range of designs.
The price of different types of bead can vary. Carol wants to estimate the cost of beads for each necklace in the designs shown. In order to do this she needs to work out how many of each type of bead is used to make each necklace.

She works on the basis that long necklaces usually contain around 150 beads in total, whereas short necklaces usually contain around 80 beads in total.

Here are her workings out for Design A.

Design A – Long, around 150 beads

**a)** Describe in words any patterns you can see in the beads.

**b)** Describe where you can see the numbers she has recorded within the picture.

**c)** Explain how you think she has come up with her final figures of 21, 84 and 42.

**d)** Short necklaces are usually made up of around 80 beads in total. Use Carol’s method to work out how many of each type of bead would be needed for the short version of Design A.
Don is working out how many beads he will need for a short necklace using Design B.

He prefers to use a ratio table. Here are his workings for Design B:

<table>
<thead>
<tr>
<th></th>
<th>Rectangle</th>
<th>Links</th>
<th>Oval</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOTAL</td>
<td>6</td>
<td>60</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>40</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

a) Describe in words any patterns you can see in this necklace.

b) Describe where you can see the numbers 1, 4, 1 and 6 within the picture.

c) How has Don made the other columns in his ratio table?

d) Don wanted to find out how many beads he would need for a short version of Design B. Copy and complete his ratio table.

**Turn to pages 23–24 in your workbook and do Workbook exercise 3.3.**
Unthreaded necklaces

Two new necklaces from Don and Carol’s children’s range arrive in their bags unthreaded. These are shown below:

Carol carefully counts the number of each type of bead within each bag.
Bag F contains 30 pink beads and 20 green beads.
Bag G contains 8 blue beads, 12 white beads and 16 lilac beads.
Carol spreads the beads out and tries to work out what each necklace could look like. She knows each one will be based on a repeating pattern.

**a)** See if you can come up with a possible repeating pattern for the beads in bag F.

**b)** See if you can come up with a possible repeating pattern for the beads in bag G.

Turn to pages 25–27 in your workbook and do Workbook exercise 3.4.
Pablo’s pizza parlour

Kate and Pam have been friends since primary school. They both work in town and regularly meet up for a lunchtime pizza at one of Pablo’s restaurants. The lunchtime menu offers large pizzas, ideal for sharing.

One lunchtime they order the ham and pineapple pizza shown above and request that it is cut into eight slices. Kate eats five of the slices, leaving three slices for Pam.

When they get the bill, Kate and Pam like to make sure that each of them pays for what they have eaten.

a) Draw a picture to show how to share out the pizza.

b) Work out how much each person should pay.
Kate and Pam decide it is much easier to cut up a rectangular pizza than a round one. So the next week they order the rectangular cheese and tomato pizza shown here. This pizza costs £7.20.

Kate thinks she will want about five pieces, while Pam only wants four. So they decide to cut the pizza into nine equal slices.

a) Draw a picture to show how to share out the pizza.

b) Share the cost of the pizza between Kate and Pam in the ratio 5 : 4.
Here is a copy of the menu for Pablo’s pizza parlour.

**Pablo’s Pizzaria**

*Made for sharing*

**Rectangular Pizzas**

- Cheese & Tomato: £7.20
- Ham & Pineapple: £7.60
- Pepperoni: £9.50
- Vegetarian: £10.20
- Seafood: £10.50
- Meat Feast: £10.80

a) If you and the person next to you were to share one of these pizzas, decide between you which one you would choose.

b) How would you cut it up?

c) When Kate and her friends go to this restaurant, it is always Kate who works out how to split the bill. Below you can see her calculations on a serviette:

```
£9.50
K
K
P
P
P
= 3.80
£1.90
= 5.70
```

Describe what these calculations tell you about the trip to the restaurant.

d) Which pizza did they have?

Turn to pages 28–29 in your workbook and do Workbook exercise 3.5.
Using the bar model for ratio problems

A teacher was marking test papers for her Year 10 class. One of the questions was:

**Share £180 in the ratio 5 : 4**

A number of the class had left this question out.

Below you see the strategy used by one of the students:

a) Describe the order in which this student may have written things down.

b) Compare this strategy with the method Kate used on her serviette in question 10.

c) In both cases a rectangular bar has been drawn. This is known as the ‘bar model’. Use a similar method by drawing a rectangular bar to share out a giant, 120cm length, block of chocolate in the ratio 3 : 5.

Turn to pages 30–32 in your workbook and do Workbook exercise 3.6.
Who is the fastest of them all?

Over the years scientists have measured and recorded the top speeds achieved by various animals.

Look at the list of animals below and try to rank them in order of speed, fastest first.

- Elephant
- Human
- Grizzly bear
- Cheetah
- Greyhound
- Snail
- Lion
- Hare
- Giant tortoise
- Horse

a) Give some reasons why it is not possible to line these animals up and race them against each other.

b) Make some suggestions as to how you could measure the speeds of the various animals.
The following table shows some recordings made by scientists.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elephant</td>
<td>300 m</td>
<td>27 s</td>
</tr>
<tr>
<td>Human</td>
<td>22 m</td>
<td>1.8 s</td>
</tr>
<tr>
<td>Horse</td>
<td>500 m</td>
<td>25 s</td>
</tr>
<tr>
<td>Greyhound</td>
<td>400 m</td>
<td>22 1/2 s</td>
</tr>
<tr>
<td>Snail</td>
<td>12 m</td>
<td>15 min</td>
</tr>
<tr>
<td>Lion</td>
<td>400 m</td>
<td>18 s</td>
</tr>
<tr>
<td>Cheetah</td>
<td>160 m</td>
<td>5 s</td>
</tr>
<tr>
<td>Hare</td>
<td>260 m</td>
<td>12 s</td>
</tr>
<tr>
<td>Grizzly bear</td>
<td>400 m</td>
<td>30 s</td>
</tr>
<tr>
<td>Giant tortoise</td>
<td>136 m</td>
<td>30 min</td>
</tr>
</tbody>
</table>

This table can help you decide who was right when you put the animals in order of speed.

a) Were you right with your 1st and 2nd place animals?
b) Were you right with your last and next to last animals?

Patricia wanted to find out who was faster between the horse and the greyhound. This is what she did:

\[
\begin{array}{c|c|c}
\text{Horse} & \text{Metres} & \text{Seconds} \\
\hline
 & 500 & 25 \\
 & 1000 & 50 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
\text{Greyhound} & \text{Metres} & 400 & 800 & 200 & 1000 \\
\hline
 & \text{Seconds} & 22\frac{1}{2} & 45 & 11\frac{1}{4} & 56\frac{1}{4} \\
\end{array}
\]

a) Describe what Patricia has done.
b) Explain how this helps Patricia to decide which animal is fastest.
Frank does the following work to help him compare the horse and the greyhound.

<table>
<thead>
<tr>
<th>Horse</th>
<th>Metres</th>
<th>500</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs</td>
<td>25</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Greyhound</th>
<th>Metres</th>
<th>400</th>
<th>200</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs</td>
<td>22 1/2</td>
<td>11 1/4</td>
<td>$5 \frac{1}{2} + \frac{1}{8} = 5 \frac{5}{8}$</td>
<td></td>
</tr>
</tbody>
</table>

**a)** Describe what Frank has done.

**b)** Explain how this helps Frank to decide which animal is fastest.

17 Choose some of the animals you were not sure about and use ratio tables to compare their speeds.

18 Find a way to compare the hare and the tortoise. How much faster is the hare than the tortoise?
World record speeds and beyond?

The table shows animals recorded over distances in which they achieved their maximum speed. So, for example, the human speed was measured over 22 metres of a 100 m sprint. The lion and the elephant were measured over distances when they were in the act of charging.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elephant</td>
<td>300 metres</td>
<td>27 s</td>
</tr>
<tr>
<td>Human</td>
<td>22 metres</td>
<td>1.8 s</td>
</tr>
<tr>
<td>Horse</td>
<td>500 metres</td>
<td>25 s</td>
</tr>
<tr>
<td>Greyhound</td>
<td>400 metres</td>
<td>22 1/2 s</td>
</tr>
<tr>
<td>Snail</td>
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<td>15 min</td>
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<tr>
<td>Lion</td>
<td>400 metres</td>
<td>18 s</td>
</tr>
<tr>
<td>Cheetah</td>
<td>160 metres</td>
<td>5 s</td>
</tr>
<tr>
<td>Hare</td>
<td>260 metres</td>
<td>12 s</td>
</tr>
<tr>
<td>Grizzly bear</td>
<td>400 metres</td>
<td>30 s</td>
</tr>
<tr>
<td>Giant tortoise</td>
<td>136 metres</td>
<td>30 min</td>
</tr>
</tbody>
</table>

Frank wonders how the human speed compares with the world record for running 100 metres.

He starts by converting the human speed into metres per second:

\[
\begin{array}{c|c|c|c|c}
\text{Distance} & 22 & 220 & 110 & 12 and a bit \\
\text{Time (s)} & 1.8 & 18 & 9 & 1 \\
\end{array}
\]

a) Describe what Frank has done and how he got his figures.

b) Frank is quite surprised by the results for the human. He knows that the world record for sprinting 100 metres is just under 10 seconds. He thinks this is like doing 10 metres in 1 second. The calculation he did above says a human can run around 12 metres in 1 second.

How can you explain this difference?

20 Draw your own ratio tables to help you answer the following questions:

a) Convert the elephant’s speed into metres per second. Do the same for the grizzly bear. Who would win that race?

b) How much faster is a horse than a greyhound in metres per second?

c) Compare the lion and the hare’s speed in metres per second. What does this tell you?
Animals on the motorway

Patricia wonders how the animal speeds compare with the speed of a car. She starts with the fastest animal, the cheetah, and tries to convert this speed into miles per hour.

\[
\begin{array}{c|c|c|c|c|c}
\text{Metres} & 160 & 320 & 1920 & 115200 & 115.2\; \text{km} \\
\text{Secs} & 5 & 10 & 60 = \frac{60\text{min}}{1\text{hr}} & 60\text{min} = \frac{60\text{min}}{1\text{hr}} & 1\text{hr}
\end{array}
\]

a) Describe what Patricia has done so far.
b) 5 miles is roughly 8 km. Use this fact to try to convert 115.2 km per hour into miles per hour.
c) Do you think a cheetah’s top speed would look out of place on a motorway?

\[\text{Cheetah}\]

\[\text{Metres}\]
\[\text{Secs}\]
\[160\]
\[5\]
\[320\]
\[10\]
\[1920\]
\[60\]
\[60\text{min}=\frac{60\text{min}}{1\text{hr}}\]
\[115200\]
\[60\text{min}=\frac{60\text{min}}{1\text{hr}}\]
\[115.2\; \text{km}\]
\[1\text{hr}\]

22 a) Choose three other animals from the table and convert their speeds into miles per hour.
b) How much faster is the hare than the tortoise in miles per hour?
c) What about the human speed in miles per hour?
d) Do you think miles per hour is a good way to look at animal speed or not?
Jewellery prices

Carol and Don want to expand their business even further. They start looking into selling some of their necklaces in Europe and in America. They wonder how much to mark up the prices for selling in Europe and in America.

a) Why might they need to charge more for their necklaces in Europe and in America compared with the UK?

b) This necklace retails at £9 in the UK.

Make some suggestions about what price you would put on this necklace:

i) in Europe
ii) in America

(The exchange rates at the time are: £1 = 1.13 Euros; £1 = 1.63 US dollars.)

c) Carol uses a ratio table to help her think about the price. This can be seen below:

\[
\begin{array}{c|ccc}
\text{£} & 1 & 10 & 9 \\
\text{Euro} & 1.13 & 11.3 & 10.17 \\
\end{array}
\]

\[
\begin{array}{c|ccc}
\text{£} & 1 & 10 & 9 \\
\text{US dollars} & 1.63 & 16.3 & 14.67 \\
\end{array}
\]

Describe what Carol has done.

d) Carol decides to price the necklace at 12 Euros for the European market and 16 dollars for the American market.

How do these prices compare with your suggestions from part b)?
24 The necklace shown below retails at £22 in the UK.

![Necklace image]

a) Suggest what price to charge in Europe.

b) Suggest what price to charge in America.

25 Carol starts looking in other shops to see what the difference is in prices in the UK compared with other countries.

Here is an example of a ticket price in the UK:

![Ticket image]

a) How do the prices shown on this ticket compare with the exchange rate of £1 = 1.13 Euros?

b) This item was made by a British company and then sold in other countries in Europe. Based on the exchange rate of £1 = 1.13 Euros, this item has been marked up for the European market. By how much has it been marked up?

c) Why might companies mark up their prices when they sell products abroad?
The following price tickets are for a different company:

\[ \begin{array}{c|c}
\text{Pounds} & \text{Euros} \\
9.99 & 12.95 \\
14.99 & 19.95 \\
19.99 & 24.95 \\
\end{array} \]

a) Are the items shown above more expensive to buy in pounds or in Euros?

b) Which ticket is showing the biggest mark-up?
Buying electrical goods in America

John is going to America on business. He has heard that some electrical goods are cheaper over there than in the UK. John would like a new iPod. He does some research on the internet, looking at the price of iPods in the UK and in America.

<table>
<thead>
<tr>
<th>Product</th>
<th>US retail price</th>
<th>UK retail price</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPod Nano</td>
<td>$149</td>
<td>£131</td>
</tr>
<tr>
<td>iPod Shuffle</td>
<td>$49</td>
<td>£40</td>
</tr>
<tr>
<td>iPod Touch – 8 GB</td>
<td>$229</td>
<td>£193</td>
</tr>
<tr>
<td>iPod Touch – 32 GB</td>
<td>$299</td>
<td>£254</td>
</tr>
<tr>
<td>iPod Touch – 64 GB</td>
<td>$399</td>
<td>£336</td>
</tr>
</tbody>
</table>

On the day John looks at these prices, the exchange rate is: £1 = $1.59.

a) John looks at these figures and does a quick calculation in his head. He reckons that it will be cheaper to buy these items in America. What do you think he did?

b) Work out how much he could save on each item if he bought it in America.

c) Use the internet to research the prices of some goods that you are interested in. Compare the prices in the UK, America and Europe to find out where they are cheapest.
Summary

Equivalent ratios

In this chapter you have looked at various ways of writing ratios.

In this necklace you can see a repeating pattern of black, red, brown, brown, brown; black, red, brown, brown, brown.

The ratio of black : red : brown is 1 : 1 : 3.

It is also possible to see a black : red : brown ratio of 2 : 2 : 6.

There are several other ways of seeing the ratio, such as 8 : 8 : 24.

The ratios:
- 8 : 8 : 24
- 2 : 2 : 6
- 1 : 1 : 3

are all equivalent ratios.

1 : 1 : 3 is the simplified ratio of 8 : 8 : 24.

Sharing in a given ratio

You learned how to use the rectangular bar model to share out quantities in a particular ratio.

In the problem below, Kate and Pam were sharing the cost of their £9.50 pizza in the ratio 2 : 3.

Kate first drew a rectangular block.

She split it into five parts (two parts and three parts).

She then figured out the cost for each part and worked out a cost for two parts and for three parts.
Using a ratio table

You saw several situations where a ratio table could help you solve a problem.

The ratio table below was used to work out how many of each type of bead would be needed in a necklace of 150 beads in total.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>5</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Links</td>
<td>4</td>
<td>40</td>
<td>80</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Oval</td>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>60</td>
<td>120</td>
<td>30</td>
<td>150</td>
</tr>
</tbody>
</table>

The ratio table below was used to see how far a greyhound doing 400 metres in 22 1/2 seconds would go in 1 second.

<table>
<thead>
<tr>
<th></th>
<th>400</th>
<th>800</th>
<th>160</th>
<th>17 and a bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metres</td>
<td>22 1/2</td>
<td>45</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

A ratio table is a really helpful tool to solve many problems in many topic areas, not just those that obviously mention ratios.