HODDER GIBSON
Model Paper WITH ANSWERS
National 5 Maths
Total marks — 40

You may NOT use a calculator.

Attempt ALL questions.

Use blue or black ink. Pencil may be used for graphs and diagrams only.

Write your working and answers in the spaces provided. Additional space for answers is provided at the end of this booklet. If you use this space, write clearly the number of the question you are attempting.

Square-ruled paper is provided at the back of this booklet.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Before leaving the examination room you must give this booklet to the Invigilator. If you do not, you may lose all the marks for this paper.
FORMULAE LIST

The roots of \( ax^2 + bx + c = 0 \) are \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Sine rule: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule: \( a^2 = b^2 + c^2 - 2bc \cos A \) or \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)

Area of a triangle: \( A = \frac{1}{2} ab \sin C \)

Volume of a sphere: \( V = \frac{4}{3} \pi r^3 \)

Volume of a cone: \( V = \frac{1}{3} \pi r^3 h \)

Volume of a pyramid: \( V = \frac{1}{3} Ah \)

Standard deviation: \( s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2/n}{n-1}} \), where \( n \) is the sample size.
1. Evaluate \( \frac{1}{3} + \frac{5}{6} \) of \( \frac{2}{5} \).  

2. Solve the inequality \( 5 - x > 2(x + 1) \).  

3. Factorise \( 2p^2 - 5p - 12 \).
4. The temperature, in degrees Celsius, at mid-day in a seaside town and the sales, in pounds, of umbrellas are shown in the scattergraph below. A line of best fit has been drawn.

(a) Find the equation of the line of best fit.  

(b) Use your answer to part (a) to predict the sales for a day when the temperature is 30 degrees Celsius.

Total marks 4
5. The diagram below represents a sphere.

\[ 6 \text{ cm} \]

The sphere has a diameter of 6 centimetres. Calculate its volume.

Take \( \pi = 3.14 \).

6. Solve algebraically the system of equations

\[
2x - 5y = 24 \\
7x + 8y = 33.
\]
7. Coffee is sold in regular cups and large cups.
   The two cups are mathematically similar in shape.

![Diagram of two cups](image)

The regular cup is 14 centimetres high and hold 160 millilitres.
The large cup is 21 centimetres high.

Calculate how many millilitres the large cup holds.

8. (a) Show that the standard deviation of 1, 1, 1, 2 and 5 is \( \sqrt{3} \).

(b) **Write down** the standard deviation of 101, 101, 101, 102 and 105.

**Total marks** 4
9. Cleano washing powder is on a special offer.

Each box on special offer contains 20\% more powder than the standard box.
A box on special offer contains 900 grams of powder.
How many grams of powder does the standard box hold?  3

10. The graph shown below has an equation of the form \( y = \cos(x - a)^\circ \).

Write down the value of \( a \).  1
11. Express $\frac{12}{\sqrt{2}}$ with a rational denominator.

Give your answer in its simplest form.

12. Each day, Marissa drives 40 kilometres to work.

(a) On Monday, she drives at a speed of $x$ kilometres per hour.

Find the time taken, in terms of $x$, for her journey.

(b) On Tuesday, she drives 5 kilometres per hour faster.

Find the time taken, in terms of $x$, for this journey.

(c) Hence find an expression, in terms of $x$, for the difference in times of the two journeys.

Total marks 5
13. William Watson's fast Foods use a logo based on parts of three identical parabola.

This logo is represented in the diagram below.

The first parabola has turning point P and equation \( y = (x + 2)^2 - 16 \).

(a) State the coordinates of P.  

(b) If R is the point (2,0), find the coordinates of Q, the minimum turning point of the second parabola. 

(c) Find the equation of the parabola with turning point S. 

Total marks 5
Duration — 1 hour and 30 minutes

Total marks — 50

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1. The National Debt of the United Kingdom was recently calculated as £1 157 818 887 139.
Round this amount to four significant figures.

2. The diagram shows vectors $s$ and $t$.

Find the components of $s + t$. 

3. The diagram below shows the graph of \( y = -x^2 \).

The point \((-3, k)\) lies on the graph.

Find the value of \( k \).
4. A health food shop produces cod liver oil capsules for its customers.

Each capsule is in the shape of a cylinder with hemispherical ends as shown in the diagram below.

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15 mm

23 mm

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The total length of the capsule is 23 millimetres and the length of the cylinder is 15 millimetres.

Calculate the volume of one cod liver oil capsule.
5. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).
Q is the midpoint of face CBFG.
Write down the coordinates of G and Q.

6. Express in its simplest form \( y^8 \times (y^3)^{-2} \).

7. A straight line is represented by the equation \( 2y + x = 6 \).
   (a) Find the gradient of this line.

   (b) Write down the coordinates of the point where this line crosses the y-axis.

Total marks 3
8. A pet shop manufactures protective dog collars.
   In the diagram below the shaded area represents one of these collars.

   AB and CD are arcs of the circles with centres at O.
   The radius, OA, is 10 inches and the radius, OC, is 18 inches.
   Angle AOB is 160°.
   Calculate the area of the collar.

9. Show that the equation \(x(5 - 2x) = 7\) has no real roots.
10. In triangle PQR

- PQ = 5 centimetres
- PR = 6 centimetres
- Area of triangle PQR = 12 square centimetres
- Angle QPR is **obtuse**.

Calculate the size of angle QPR.  

11. AD is a diameter of a circle, centre O.

B is a point on the circumference of the circle.

The chord BD is extended to a point C, outside the circle.

Angle BOA = 98°.

DC = 9 centimetres.

The radius of the circle is 7 centimetres.

Calculate the length of AC.  

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12. A right-angled triangle has dimensions, in centimetres, as shown.

\[ \begin{array}{c}
\text{x} \\
\text{x + 8} \\
\text{x + 7}
\end{array} \]

**Calculate** the value of \( x \).

5

13. For reasons of safety, a building is supported by two wooden struts, represented by DB and DC in the diagram below.

\[ \begin{array}{c}
\text{D} \\
\text{A} \\
\text{B} \\
\text{C}
\end{array} \]

\[ \begin{array}{c}
\text{Angle ABD = 55°.} \\
\text{Angle BCD = 38°.} \\
\text{BC is 5 metres.} \\
\text{Calculate the height of the building represented by AD.}
\end{array} \]

5
14. Due to the threat of global warming, scientists recommended in 2010 that the emissions of greenhouse gases should be reduced by 50% by the year 2050.

The government decided to reduce the emissions of greenhouse gases by 15% every ten years, starting in the year 2010.

Will the scientists’ recommendations have been achieved by 2050?

You must give a reason for your answer.

15. The depth of water, $D$ metres, in a harbour is given by the formula

$$D = 3 + 1.75 \sin 30h$$

where $h$ is the number of hours after midnight.

(a) Calculate the depth of the water at 5 am.

(b) Calculate the maximum difference in depth of the water in the harbour.

Do not use a trial and improvement method.

Total marks 4

[END OF MODEL PAPER]
1. $3^\frac{1}{2}$
2. $x < 1$
3. $(2p + 3)(p - 4)$
4. (a) $S = -4T + 130$
   (b) £10
5. $113.04$ cm$^3$
6. $x = 7$, $y = -2$
7. $540$ millilitres
8. (a) $\sqrt{\frac{(1 - 2)^3 + (1 - 2)^2 + (1 - 2)^2 + (1 - 2)^2 + (1 - 2)^2}{5 - 1}} = \sqrt{\frac{15}{4}} = \sqrt{3}$
   or $\sqrt{\frac{(1 + 1 + 1 + 1 + 1)^2}{5 - 1}} = \sqrt{\frac{5^2}{4}} = \sqrt{3}$
   (b) $\sqrt{3}$
9. $750$ grams
10. $45^\circ$
11. $6\sqrt{2}$
12. (a) $\frac{40}{x}$
   (b) $\frac{40}{x + 5}$
   (c) $\frac{200}{x(x + 5)}$
13. (a) $(-2, -16)$
    (b) $(6, -16)$
    (c) $y = (x - 14)^2 - 16$
14. £1 158 000 000 000
15. (a) $\left(\frac{5}{4}\right)$
16. $-9$
17. $1022$ mm$^3$
18. $G(0, 2, 2), Q(1, 2, 1)$
19. $y^2$
20. (a) $-\frac{1}{2}$
    (b) $(0, 3)$
21. $313$ in$^2$
22. discriminant = $-31$; no real roots since discriminant < 0.
23. $126.9^\circ$
24. $21$ centimetres
25. $5$
26. $8.6$ metres
27. $0.85^4 = 0.522$; no, since $0.522 > 0.5$
28. (a) $3.875$ metres
    (b) $3.5$ metres