Section 1 Algebra

Target your revision (Chapters 1–4) (pages 1–2)

1. \[2(x - 3y) + 3(y + 1) + x\]
   \[= 2x - 6y + 3y + 3 + x\]
   \[= 3x - 3y + 3\]

2. \[\sqrt{18} + \sqrt{32} = 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}\]

3. \[\frac{3}{2 - \sqrt{3}} = \frac{3}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}\]
   \[= \frac{6 + 3\sqrt{3}}{4 - 3}\]
   \[= 3(2 + \sqrt{3})\]

4. (i) \[f(x) + g(x) = x^3 - x^2 + x - 1 + 3x + 1\]
   \[= x^3 - x^2 + 4x\]
   (iii) \[f(x) \times g(x) = (x^3 - x^2 + x - 1)(3x + 1)\]
   \[= 3x^4 - 3x^3 + 3x^2 - 3x + x^3 - x^2 + x - 1\]
   \[= 3x^4 - 2x^3 + 2x^2 - 2x - 1\]

5. \[3(x - 2) - 1 = 1 + x\]
   \[\Rightarrow 3x - 6 - 1 = 1 + x\]
   \[\Rightarrow 2x = 8\]
   \[\Rightarrow x = 4\]

6. \[2x^2 - x - 3 = 0\]
   \[\Rightarrow (2x - 3)(x + 1) = 0\]
   \[\Rightarrow x = -1, \frac{3}{2}\]

7. \[2x^2 + x - 5 = 0\]
   \[\Rightarrow x = \frac{-1 \pm \sqrt{1 + 40}}{4} = \frac{-1 \pm 41}{4}\]
   \[= 1.351, -1.851\]

8. \[x^2 - x + 7 = 0\]
   Discriminant \(b^2 - 4ac = -27 < 0\)

9. \[n^2 - 6n + 15 = (n - 3)^2 - 9 + 15\]
   \[= (n - 3)^2 + 6 \geq 6\] for all \(n\)
10

\[ y = \frac{1}{x} \]

\[ x = -1.6, 3.6 \]

11 \( f(x) = 3x^2 + 4x - 4 \Rightarrow f(-2) = 12 - 8 - 4 = 0 \)
So \((x + 2)\) is a factor of \(f(x)\).

12 \( f(x) = x^3 - 2x^2 - 5x + 6 = 0 \)
\( f(1) = 1 - 2 - 5 + 6 = 0 \)
\( f(3) = 27 - 18 - 15 + 6 = 0 \)
\( f(-2) = -8 - 8 + 10 + 6 = 0 \)
\( \Rightarrow f(x) = (x - 1)(x - 3)(x + 2) = 0 \)
\( \Rightarrow x = 1, 3, -2 \)

13 \( 2x + 5y = 19 \)
\( 6x - y = 9 \)
Multiply first equation by 3.
\( 6x + 15y = 57 \)
\( 6x - y = 9 \)
Subtract
\( 16y = 48 \Rightarrow y = 3 \Rightarrow x = 2 \)

14 \( y = x^2 - 3x + 5 \) and \( y = 5x - 7 \)
Substitute
\( x^2 - 3x + 5 = 5x - 7 \)
\( \Rightarrow x^2 - 8x + 12 = 0 \)
\( \Rightarrow (x - 2)(x - 6) = 0 \)
\( \Rightarrow x = 2 \) giving \( y = 3 \)
and \( x = 6 \) giving \( y = 23 \)

15 Let John's age last year be \( x \).
Then Paul's age last year was \( 2x \).
This year they are \( x + 1 \) and \( 2x + 1 \) respectively.
\( x + 1 + 2x + 1 = 77 \)
\( 3x = 75 \)
\( x = 25 \)
John's age this year is 26 and Paul's age is 51.

16 \( 3 < 2x - 1 < 19 \)
\( \Rightarrow 4 < 2x < 20 \)
\( \Rightarrow 2 < x < 10 \)
Chapter 1 Algebraic manipulation

1.1 Manipulating algebraic expressions

Exam-style question (page 5)

\[
\frac{2 + \frac{x-3}{5}}{x} = \frac{2 \times 5 + (x-3) \times x}{x \times 5} = \frac{10 + x(x-3)}{5x} = \frac{x^2 - 3x + 10}{5x}
\]

Find the common denominator which in this case is 5x.

Add the two fractions which is done because they have the same denominator.

Tidy up the numerator. In this case the quadratic function cannot be factorised so this is the simplest form.

1.2 Manipulating expressions involving square roots

Exam-style question (page 7)

\[
\frac{4 - \sqrt{2}}{4 + \sqrt{2}} = \frac{4 - \sqrt{2}}{4 + \sqrt{2}} \times \frac{4 - \sqrt{2}}{4 - \sqrt{2}} = \frac{(4 - \sqrt{2})^2}{4^2 - (\sqrt{2})^2} = \frac{16 - 8\sqrt{2} + 2}{16 - 2} = \frac{18 - 8\sqrt{2}}{14} = \frac{9 - 4\sqrt{2}}{7}
\]

As before, multiply top and bottom of the fraction by the number which rationalises the denominator. If the denominator is of the form \(a + \sqrt{b}\) then the number to be used will be \(a - \sqrt{b}\).

Simplify the denominator and numerator.

Note that a factor 2 can be cancelled from the top and bottom lines.

\[\text{Define } P \text{ as population this year and } P_n \text{ as population } n \text{ years from now.}
\]

\[P_n = P \times 1.05^n\]

\[P_{10} = 10000 \times 1.05^{10} = 16300 \text{ (to 3 s.f.)}\]
Chapter 2 Polynomials

2.1 Polynomials
Exam-style question (page 9)

(i) \[ x^3 + 4x - 3 + 2(x-2)(x+1) \]
\[ = x^3 + 4x - 3 + 2(x^2 - x - 2) \]
\[ = x^3 + 4x - 3 + 2x^2 - 2x - 4 \]
\[ = x^3 + 2x^2 + 2x - 7 \]

(ii) \[ \frac{x^2 + x + 1}{(x+1)(x^2 + 2x + 5)} \]
\[ = \frac{x^2 + x}{x^2 + 2x} \]
\[ = \frac{x + 5}{x + 1} \]
\[ = \frac{4}{4} \]

2.2 Review of solving linear equations
Exam-style question (page 11)

\[ \frac{2(x-1)}{3} - 2 = \frac{3x+1}{4} \]
\[ \Rightarrow \frac{2(x-1) \times 12}{3} - 2 \times 12 = \frac{(3x+1) \times 12}{4} \]
\[ \Rightarrow \frac{2(x-1) \times 12}{3} - 2 \times 12 = \frac{3(3x+1)}{4} \]
\[ \Rightarrow 8(x-1) - 24 = 3(3x+1) \]
\[ \Rightarrow 8x - 8 - 24 = 9x + 3 \]
\[ \Rightarrow -x = 8 + 24 + 3 = 35 \]
\[ \Rightarrow -x = 35 \Rightarrow x = -35 \]

2.3 Solving quadratic equations
Exam-style question (page 13)

(i) \[ x^2 - 3x - 1 = \left( x^2 - 3x + \frac{9}{4} \right) - \left( \frac{3}{2} \right)^2 - 1 \]
\[ \Rightarrow \left( x - \frac{3}{2} \right)^2 = \frac{13}{4} \]

(ii) \[ x^2 - 3x - 1 = 0 \]
\[ \Rightarrow \left( x - \frac{3}{2} \right)^2 = \frac{13}{4} \]
\[ \Rightarrow x - \frac{3}{2} = \pm \sqrt{\frac{13}{4}} = \pm \frac{1}{2} \sqrt{13} \]
\[ \Rightarrow x = \frac{3}{2} \pm \frac{1}{2} \sqrt{13} \]
\[ = 3.303 \text{ or } -0.303 \]
2.4 Solving cubic equations
Exam-style question (page 14)

(i) \( f(-1) = -1 - 5 - 2 + 8 = 0 \)

(ii) From (i), \((x + 1)\) is a factor.

\[ f(x) = (x + 1)(x^2 - 6x + 8) \]
\[ = (x + 1)(x - 2)(x - 4) = 0 \]
\[ \Rightarrow x = -1, 2 \text{ or } 4 \]

Chapter 3 Applications of equations and inequalities in one variable

3.1 Review of simultaneous equations
Exam-style question (page 16)

\[ y = x + 6 \text{ and } y = x^2 - x + 3 \]
\[ \Rightarrow x + 6 = x^2 - x + 3 \]
\[ \Rightarrow x^2 - 2x - 3 = 0 \]
\[ \Rightarrow x = 3 \text{ or } x = -1 \]
\[ \Rightarrow x = -1 \text{ giving } y = -1 + 6 = 5 \]

3.2 Setting up equations
Exam style question (page 19)

(i) Gavin’s time = \( \frac{140}{v} \)
Simon’s time = \( \frac{140}{v+5} \)

(ii) The difference in times is 15 minutes which is \( \frac{1}{4} \) hr.

\[ \frac{140}{v} - \frac{140}{v+5} = \frac{1}{4} \]
\[ \Rightarrow 140(v+5) - 140v = v(v+5) \]
\[ \Rightarrow 560 = v(v+5) \]

(iii) \( v^2 + 5v - 2800 = 0 \)
\[ \Rightarrow v = \frac{-5 \pm \sqrt{25 + 11200}}{2} = \frac{-5 \pm \sqrt{11225}}{2} \]
\[ = v = 50.47 \]

(Reject the negative answer.)

So, Gavin’s speed is 50.47 km per hour and so his time is 2.77 hours = 2 hours 46 minutes.
Simon’s time is 2 hours 31 minutes.
3.3 Inequalities

Exam-style question (page 21)

(i) Factorising gives \(x^2 + 2x - 15 = (x + 5)(x - 3)\)

For \(x^2 + 2x - 15 \geq 0\)

\((x + 5)(x - 3) \geq 0\)

Since the product of the factors \((x + 5)\) and \((x - 3)\) is \(\geq 0\)

Either Both factors are \(\geq 0\)

Or Both factors are \(\leq 0\)

Either Both \((x + 5) \geq 0\) and \((x - 3) \geq 0\)

\(\Rightarrow x \geq -5\) and \(x \geq 3\) and so \(x \geq 3\)

Or Both \((x + 5) \leq 0\) and \((x - 3) \leq 0\)

\(\Rightarrow x \leq -5\) and \(x \leq 3\) and so \(x \leq -5\)

\(x\) satisfies \(x \geq 3\) or \(x \leq -5\)

(ii) The graph shows the solution for the inequality.

Chapter 4 Recurrence relationships

4.1 First order recurrence relationships

Exam-style question (page 24)

(i) \(L_{n+1} = L_n \times 1.1\)

(ii) At the end of the first year the interest is \(\£2000 \times \frac{10}{100} = \£200\).

He must therefore pay at least \(\£200\) per year.

(iii) The loan becomes \(\£2000 \times (1.1)^5 = \£3221.02\)

4.2 Second order recurrence relationships

Exam-style question (page 25)

(i) \(u_1 = 1\)

\(u_2 = 2\)

\(u_3 = 4u_2 - 3u_1 = 4 \times 2 - 3 \times 1 = 5\)

\(u_4 = 4u_3 - 3u_2 = 4 \times 5 - 3 \times 2 = 14\)

\(u_5 = 4u_4 - 3u_3 = 4 \times 14 - 3 \times 5 = 41\)
(ii) \[ u_1 = \frac{1}{2}(1+3^0) = 1 \]
\[ u_2 = \frac{1}{2}(1+3^1) = 2 \]
\[ u_3 = \frac{1}{2}(1+3^2) = 5 \]
\[ u_4 = \frac{1}{2}(1+3^3) = 14 \]
\[ u_5 = \frac{1}{2}(1+3^4) = 41 \]

Amit is right.
His rule gives the correct answers for the first 5 terms.

**Review questions (Chapters 1–4)**
*(page 26)*

1. \[(x-3)(x+2)+(x+3)(x+2)\]
   \[= (x+2)(x-3+x+3)\]
   \[= 2x(x+2)\]

2. \[(\sqrt{3}+3\sqrt{5})(2\sqrt{3}-\sqrt{5}) = 6-\sqrt{15}+6\sqrt{15}-15\]
   \[= -9+5\sqrt{15}\]

3. \[\frac{1}{2+\sqrt{2}} + \frac{1}{2-\sqrt{2}}\]
   \[= \frac{2-\sqrt{2}+2+\sqrt{2}}{(2+\sqrt{2})(2-\sqrt{2})}\]
   \[= \frac{4}{4-2}\]
   \[= 2\]

4. \[\frac{3\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}\]
   \[= \frac{3\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}} \times \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-\sqrt{2}}\]
   \[= \frac{(3\sqrt{3}-\sqrt{2})(2\sqrt{3}-\sqrt{2})}{(2\sqrt{3})^2-(\sqrt{2})^2}\]
   \[= \frac{6\times3-3\sqrt{6}-2\sqrt{6}+2}{12-2}\]
   \[= \frac{20-5\sqrt{6}}{10}\]
   \[= 2-\frac{1}{2}\sqrt{6}\]

So \(a = 2, \ b = \frac{1}{2}, \ n = 6\)
\[ \frac{x^2 - 3x + 9}{x + 2} \left( \frac{x^3 - x^2 + 3x + 4}{x + 2} \right) \]

\[ \frac{x^3 - x^2 + 3x + 4}{x + 2} \]

\[ \frac{x^3 + 2x^2 - 3x + x^2 - 6x}{9x + 4} \]

\[ 9x + 18 \]

So the quotient is \( x^2 - 3x + 9 \) (and the remainder is \(-14\))

\[ \frac{2(x-1)}{3} - 2 = \frac{3x+1}{4} \Rightarrow 8(x-1) - 24 = 3(3x+1) \]

\[ \Rightarrow 8x - 32 = 9x + 3 \Rightarrow x = -35 \]

\[ x^2 - 4x - 7 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16 + 28}}{2} = \frac{4 \pm \sqrt{44}}{2} \]

\[ \Rightarrow x = 2 \pm \sqrt{11} \Rightarrow x = 5.32 \text{ and } -1.32 \]

\[ \frac{3}{x-1} + \frac{2}{x-2} = 1 \]

\[ \Rightarrow 3(x-2) + 2(x-1) = (x-2)(x-1) \]

\[ \Rightarrow 5x - 8 = x^2 - 3x + 2 \]

\[ \Rightarrow x^2 - 8x = -10 \]

\[ \Rightarrow x^2 - 8x + 16 = 16 - 10 \]

\[ \Rightarrow (x - 4)^2 = 6 \]

\[ \Rightarrow x - 4 = \pm \sqrt{6} \]

\[ \Rightarrow x = 4 \pm \sqrt{6} \]

9 \[(i) \quad f(x) = x^3 - 8x^2 + 5x + 14 \]

\[ f(2) = 8 - 32 + 10 + 14 = 0 \]

So by the factor theorem \((x - 2)\) is a factor of \(f(x)\).

\[(ii) \quad \text{By inspection } f(-1) = 0 \]

\[ \Rightarrow f(x) = x^3 - 8x^2 + 5x + 14 \]

\[ = (x - 2)(x + 1)(x - 7) \]

So \(f(x) = 0 \Rightarrow x = 2, -1, 7\)

10 The roots must be factors of \(22 = 1 \times 2 \times 11\)

\[ x^3 + ax^2 + bx - 22 = 0 \]

\[ f(1) = 1 + a + b - 22 = 0 \]

\[ \Rightarrow a + b = 21 \]

\[ f(2) = 8 + 4a + 2b - 22 = 0 \]

\[ \Rightarrow 4a + 2b = 14 \]

\[ \Rightarrow 2a + b = 7 \]

Solve simultaneously

\[ \Rightarrow a = -14, b = 35 \]

The question states that they are distinct and positive.

Use the factor theorem twice.

[Stating that \(f(11) = 0\) could be used but would involve large numbers.]

Solve simultaneously.
11 \[ y = x + 3 \text{ and } y = x^2 - 2x + 5 \]
\[ \Rightarrow x + 3 = x^2 - 2x + 5 \]
\[ \Rightarrow x^2 - 3x + 2 = 0 \]
\[ \Rightarrow (x - 1)(x - 2) = 0 \]
\[ \Rightarrow x = 1 \text{ giving } y = 4 \]
and \( x = 2 \) giving \( y = 5 \)

12 Carla’s age now is \( x \). Jean’s age now is \( 4x \)
Carla’s age 4 years ago was \( x - 4 \).
Jean’s age 4 years ago was \( 4x - 4 \)
\[ \Rightarrow 4x - 4 = 6(x - 4) \]
\[ \Rightarrow 4x - 4 = 6x - 24 \]
\[ \Rightarrow 2x = 20 \]
\[ \Rightarrow x = 10 \]
Carla’s age now is 10 years.

13 (i) By day: \[ \frac{200}{v} \], by night: \[ \frac{200}{v + 20} \]
(ii) \[ \frac{200}{v} - \frac{200}{v + 20} = \frac{50}{60} \Rightarrow 200(v + 20 - v) = \frac{5}{6}v(v + 20) \]
\[ \Rightarrow 24000 = 5v^2 + 100v \Rightarrow v^2 + 20v - 4800 = 0 \]
(iii) \[ v^2 + 20v - 4800 = 0 \Rightarrow (v + 80)(v - 60) = 0 \]

14 (i) \[ 3 - x > 5(x + 1) \]
\[ \Rightarrow 3 - x > 5x + 5 \]
\[ \Rightarrow 6x < -2 \]
\[ \Rightarrow x < -\frac{1}{3} \]
(ii) \[ x^2 + 5x < 6 \Rightarrow x^2 + 5x - 6 < 0 \]
\[ \Rightarrow (x + 6)(x - 1) < 0 \]
\[ \Rightarrow -6 < x < 1 \]

15 \[ -8 < 3x - 1 < 13 \]
\[ -7 < 3x < 14 \]
\[ \frac{-7}{3} < x < \frac{14}{3} \]
The following integers are within this range:
\(-2, -1, 0, 1, 2, 3, 4\)
16 (i) If rate of interest is \( r \)% then \( 2100 = 2000(1 + r) \)
\[
\Rightarrow 1 + r = \frac{2100}{2000} = 1.05 \Rightarrow r = 5\%
\]

(ii) \( A_n = 2000 (1 + r)^n \) is the amount of money in the account after \( n \) years, so
\[
A_3 = 2000 (1.05)^3
= 2315.25
\]

17 (i) \( u_n = A \times 2^n + B \times 3^n \)
\[
n = 1 \Rightarrow 1 = 2A + 3B
\]
\[
n = 2 \Rightarrow 5 = 4A + 9B
\]
\[
2 = 4A + 6B
\]
\[
\Rightarrow 3 = 3B \Rightarrow B = 1
\]
\[
\Rightarrow A = -1
\]
i.e. \( u_n = 3^n - 2^n \)

(ii) \( u_n = 3^n - 2^n \)
\[
\Rightarrow u_3 = 27 - 8 = 19
\]
\[
u_4 = 81 - 16 = 65
\]

You could also use the original expression for the series:
\[
u_3 = (5 \times 5) - (6 \times 1) = 19
\]
\[
u_4 = (5 \times 19) - (6 \times 5) = 65
\]
Section 2 Coordinate geometry in two dimensions

Target your revision (Chapters 5–7) (page 27)

1 \[ \frac{y-3}{-2-3} = \frac{x-2}{4-2} \]
   \[ \Rightarrow \frac{y-3}{-5} = \frac{x+2}{6} \]
   \[ \Rightarrow 6y-18 = -5x-10 \]
   \[ \Rightarrow 6y+5x-8 = 0 \]

2 \[ 2x+5y = 7 \Rightarrow 5y = 7-2x \]
   \[ \Rightarrow y = -\frac{2}{5}x + \frac{7}{5} \]
   
   So gradient is \( -\frac{2}{5} \)

3 Any line parallel to \( 3x + 4y = 6 \) has equation \( 3x + 4y = c \).
   The line through \((1, 2)\) has \( 3 \times 1 + 4 \times 2 = c = 11 \)
   \[ \Rightarrow 3x + 4y = 11 \]
   
   Any line perpendicular to \( 3x + 4y = 6 \) has equation \( 4x - 3y = c \)
   (so that \( m_1 \times m_2 = -1 \))
   The line through \((1, 2)\) has \( 4 \times 1 - 3 \times 2 = c = -2 \)
   \[ \Rightarrow 4x - 3y + 2 = 0 \]

4 \[ x^2 + y^2 - 4x + 6y = 3 \]
   \[ \Rightarrow (x^2 - 4x) + (y^2 + 6y) = 3 \]
   \[ \Rightarrow (x^2 - 4x + 4) + (y^2 + 6y + 9) = 3 + 4 + 9 \]
   \[ \Rightarrow (x-2)^2 + (y+3)^2 = 16 \]
   
   So the centre has coordinates \((2, -3)\)
   and the radius is \( \sqrt{16} = 4 \)

5 \[ 2x+5y = 20 \]
6

On a hand-drawn sketch you should indicate the turning point, the intercept on the y-axis and the approximate intercepts on the x-axis.

7

\[ y = x^3 - 2x^2 - x + 1 \]

8

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Chapter 5 Points, lines and circles

5.1 Points and lines
Exam-style question (page 30)
Always start by drawing a diagram.

(i) To be a trapezium, two sides must be parallel; it looks from the diagram that these lines are AB and CD.
Gradient of AB = \( \frac{2 - 0}{2 - 1} = 2 = m_1 \)
Gradient of CD = \( \frac{1 - 3}{4 - 2} = 2 = m_2 \)
m_1 = m_2 so the lines are parallel.
So ABCD is a trapezium.

(iii) Maximum value of \( 2x + y = 13 \). (The nearest integer point to the intersection of the two lines is (4, 5).)
Chapter 5 Points, lines and circles

(iii) Gradient of BC = \(\frac{2-1}{2-4} = -\frac{1}{2} = m_3\)
Gradient of AE = \(\frac{1-0}{3-1} = \frac{1}{2} = m_4\)
\(m_3 = m_4\) so the lines are parallel.
So BC is parallel to AE.
Since AB is parallel to CE, opposite sides of ABCE are parallel.
Therefore ABCE is a parallelogram.

(iii) length \(AB = \sqrt{(2-1)^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}\)
length \(BC = \sqrt{(2-4)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5}\)
Since adjacent sides are equal in length, ABCD is a rhombus.
(iv) \(m_3 m_4 = 2x - \frac{1}{2} = -1\)
Since adjacent sides are perpendicular as well as equal, it follows that the rhombus ABCE is a square.

5.2 The circle
Exam-style question (page 32)
Always start by drawing a diagram.

(i) The equation of the circle is \((x - 0)^2 + (y - 1)^2 = 9\)
that is, \(x^2 + (y-1)^2 = 9\) or \(x^2 + y^2 - 2y - 8 = 0\)
(ii) When \(y = 0, x^2 + 1 = 9\)
\(\Rightarrow x = \pm \sqrt{8}\)
so distance = \(2\sqrt{8}\)
(iii) \(d^2 = (2-0)^2 + (3.5-1)^2 = 10.25\)
\(d^2 > 9\)
So the point is outside the circle.

Now it is necessary to prove that the other pair of lines are parallel. (A property of a parallelogram is that opposite sides are parallel.)

A parallelogram is a rhombus if adjacent sides are equal in length.

All 4 sides of a rhombus are equal. In a square, adjacent sides are also perpendicular.

Note: In an examination you should give your answer in the form demanded. If there is no direction then it will be acceptable to give a numerical answer.
Use Pythagoras to find the distance of the point from the centre.

The square of the distance is greater than 9.
Chapter 6 Graphs

6.1 The equation of a line and its graphical representation
Exam-style question (page 34)
When \( x = 0 \), \(-3y = 12\)
\[ \Rightarrow y = -4 \]
This shows that the line passes through the point (0, -4).
When \( y = 0 \), \(4x = 12\)
\[ \Rightarrow x = 3 \]
This shows that the line passes through the point (3, 0).

6.2 Plotting or sketching polynomial functions
Exam-style question (page 36)

(i) 

(ii) \[ x^3 - 6x^2 + 11x - 3 = -x^3 + 6x^2 - 11x + 9 \]
\[ \Rightarrow 2x^3 - 12x^2 + 22x - 12 = 0 \]
\[ \Rightarrow x^3 - 6x^2 + 11x - 6 = 0 \]
The factors of 6 are \( \pm1, \pm2 \) and \( \pm3 \).
\[ f(1) = f(2) = f(3) = 0 \]
\[ \Rightarrow (x - 1)(x - 2)(x - 3) = 0 \]
\[ x = 1, 2, 3 \]
6.3 Trigonometric and exponential functions
Exam-style question (page 39)

(i) The solution to the equation is the values of \( x \) where the graphs cross. An estimate is 1.2.

(ii) For \( x = 1.2 \)
\[
\begin{align*}
  y &= 2^x + 1.5 \\
  &= 3.80 \\
  &= 3.74 \\
  &= 3.80 - 3.74 \\
  &= 0.06 \\
  &\approx 0.06
\end{align*}
\]

(iii) The graph is as shown.

3.74 is close to the root.

Chapter 7 Linear inequalities in two variables

7.1 Linear inequalities in two variables
Exam-style question (page 43)

(i) Let \( x \) be the number of chairs made and \( y \) be the number of tables made.

Then the inequalities are:
\[
\begin{align*}
  45x + 50y &\leq 700, \\
  x &\geq 3y, \\
  y &> 0
\end{align*}
\]

(ii) The graph is as shown.
The objective function is \( P = 20x + 35y \).

The line \( 20x + 35y = 300 \) is drawn on the graph.

The profit is maximised by finding the value of \( P \) so that the line \( P = 20x + 35y \) passes through the point \( A \).

This point does not have integer coordinates.

The nearest points are (11, 3), (12, 3).

So the manufacturer should make 12 chairs and 3 tables giving a profit of \( 12 \times 20 + 3 \times 35 = \£345 \).

**Review questions (Chapters 5–7)**

*pages 44–45*

1. \( \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \)
   
   with \( (x_1, y_1) \) the point (1, −3)

   and \( (x_2, y_2) \) the point (2, 6)

   gives \( \frac{y - (-3)}{6 - (-3)} = \frac{x - 1}{2 - 1} \)

   \( \Rightarrow \frac{y + 3}{9} = \frac{x - 1}{1} \)

   \( \Rightarrow y + 3 = 9x - 9 \)

   \( \Rightarrow y = 9x - 12 \)

2. (i) Gradient of line \( AC = \frac{-3 - 0}{7 - (-2)} = \frac{-1}{3} \)

   Gradient of line \( BD = \frac{-4 - 2}{0 - (-2)} = \frac{3}{2} \)

   The gradients satisfy the condition \( m_1 m_2 = -1 \)

   and so are perpendicular.

   (ii) Midpoint of \( BD \) is \( \left( \frac{2 + 0}{2}, \frac{2 - 4}{2} \right) = (1, -1) \)

   Gradient of line from \( A \) to this point \( = \frac{-1 - 0}{1 - (-2)} = \frac{-1}{3} \)

   Same gradient means that \( A \) and this point are on the same line, \( AC \).

3. (i) Compare with \( y = mx + c \)

   First line: \( y = -2x + 4 \) so gradient = \(-2\)

   Second line: \( 4x + 2y = 5 \Rightarrow 2x + y = \frac{5}{2} \)

   \( \Rightarrow y = -2x + \frac{5}{2} \) so gradient = \(-2\)

   Same gradient means that the lines are parallel.
(ii) Gradient = \(-2\) means gradient of perpendicular lines = \(\frac{1}{2}\).
So any line with this gradient has equation \(y = \frac{1}{2}x + c\).
Satisfied by \((1, 6)\) \(\Rightarrow 6 = \frac{1}{2} + c\)
\(\Rightarrow c = \frac{11}{2} \Rightarrow y = \frac{1}{2}x + \frac{11}{2}\)
\(\Rightarrow 2y = x + 11\)

4 (i) \(x^2 + y^2 - 2x - 4y - 20 = 0\)
\(\Rightarrow (x - 1)^2 - 1 + (y - 2)^2 - 4 - 20 = 0\)
\(\Rightarrow (x - 1)^2 + (y - 2)^2 = 25\)
i.e. Centre \((1, 2)\), radius 5

(ii) \((x - 1)^2 + (y - 2)^2 = 25\)
\(\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 - 25 = 0\)
\(\Rightarrow 2x^2 - 2x - 24 = 0\)
\(\Rightarrow x^2 - x - 12 = 0\)
\(\Rightarrow (x - 4)(x + 3) = 0\)
\(x = 4\) giving \(y = 6\)
and \(x = -3\) giving \(y = -1\)
So points are \((-3, -1)\) and \((4, 6)\)

5 (i) \(\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}\)
with \((x_1, y_1)\) the point \((1, 2)\)
and \((x_2, y_2)\) the point \((5, 8)\)
gives \(\frac{y - 2}{8 - 2} = \frac{x - 1}{5 - 1}\)
\(\Rightarrow \frac{y - 2}{6} = \frac{x - 1}{4}\)
\(\Rightarrow 2y - 4 = 3x - 3\)
\(\Rightarrow 2y = 3x + 1\)

(ii) \(\left(\frac{1 + 5}{2}, \frac{2 + 8}{2}\right) = (3, 5)\)

(iii) The gradient of \(AC\) is \(\frac{3}{2}\).
Equation of line through \((3, 5)\) perpendicular to line with gradient \(\frac{3}{2}\)
\(y - 5 = -\frac{2}{3}(x - 3) \Rightarrow 3y - 15 = 6 - 2x\)
\(\Rightarrow 3y + 2x = 21\)
When \(x = 0\), \(y = 7\)
\(D\) is the point \((0, 7)\).
(iv) Length of DC = \(\sqrt{(3-0)^2 + (5-7)^2}\)
\[= \sqrt{3^2 + 2^2} = \sqrt{13}\]
\[
\Rightarrow \text{equation of circle is } (x-0)^2 + (y-7)^2 = 13
\]
\[
\Rightarrow x^2 + (y-7)^2 = 13
\]

6 (i) \(x^2 + y^2 - 4x - 6y - 12 = 0\)
\[
\Rightarrow (x-2)^2 - 4 + (y-3)^2 - 9 = 12
\]
\[
\Rightarrow (x-2)^2 + (y-3)^2 = 25
\]
Centre, C, (2, 3), Radius 5

(ii) \((x-2)^2 + (y-3)^2 = 25\)
Substitute: \((5-2)^2 + (7-3)^2 = 3^2 + 4^2 = 25\)
So T lies on a circle.

(iii) Gradient of CT = \(\frac{7-3}{5-2} = \frac{4}{3}\)
\[
\Rightarrow \text{Gradient of tangent at } T = -\frac{3}{4}
\]
\[
y - 7 = -\frac{3}{4}(x-5)
\]
\[
\Rightarrow 4y - 28 = 15 - 3x
\]
\[
\Rightarrow 4y + 3x = 43
\]

(iv) From T to C is ‘back 3 and down 4’
The other end of the diameter through C and T is from C ‘back 3 and down 4’ i.e. (-1, -1)
So the other tangent (parallel to the one through T) is \(4y + 3x = c\)
which is satisfied by (-1, -1)
\[
\Rightarrow 4y + 3x = -4 - 3 = -7
\]

7 (i)

(ii) (1, 2)

8 (i) Two turning points.

(ii) (-1, 8) and (3, -24)

(iii) When \(x = 4\) and \(x = -2\)

The circle has radius equal to the length of DC so that AB is a tangent.

If the other end of the diameter through T and C is T' then C is the midpoint of TT'.

The two tangents will be parallel.
9  (i) \[
\begin{array}{c|ccccccc}
\theta & 0 & 30 & 60 & 90 & 120 & 150 & 180 \\
\hline
\sin \theta & 0 & 0.5 & 0.87 & 1 & 0.87 & 0.5 & 0 \\
\cos(\theta - 30) & 0.87 & 1 & 0.87 & 0.5 & 0 & -0.5 & -0.87
\end{array}
\]
   (ii) $\theta = 60^\circ$

10  (i) Because, for any value of $a > 0$ $y = ka^x$ goes through (0, 3).
   (ii) Clear indication that the data are above the curve.
   (iii) $a > 2$

11  (i) The line $x + 2y = 11$ is shown. This is the greatest value if $x$ and $y$
     are to be non-zero integers.

12  (i) Cost: $18x + 11y \leq 200$
    Man hours: $7x + 6y \leq 84$
    Customer demand: $x \geq 2$
    and $y \geq 2$

Tabulate values of the two functions.

Look at each constraint given in the question.
(iii) Objective function is profit:
\[ P = 70x + 50y \]

The line on the graph illustrates that the maximum value of \( P \) will occur where the two lines meet.

This is not an integer point, so all points around it need to be tested.

Moving the objective function around indicates that the greatest value of \( P \) will occur at \((9, 3)\) giving 780.

Hence make 9 of component \( X \) and 3 of component \( Y \), giving a profit of £780.
Section 3 Trigonometry

**Target your revision (Chapters 8–9) (page 46)**

1. The calculator will give \( \sin^{-1} \theta = 23.578... \) which rounds to 23.6°. There is a second angle, in the 2nd quadrant, with this ratio which is \( 180° - 23.6° = 156.4° \).

2. ![Graph](image)

3. Area = \( \frac{1}{2} ch = \frac{1}{2} ca \sin B = \frac{1}{2} \times 4 \times 5 \times \sin 70 \)
   \[ = 10 \sin 70 = 9.396... \text{cm}^2 \]
   \[ = 9.40 \text{cm}^2 \text{ to 3 s.f.} \]

4. Using the cosine rule
   \[ AC^2 = AB^2 + CB^2 - 2 \times AB \times CB \times \cos 40 \]
   \[ = 16 + 25 - 40 \cos 40 \]
   \[ = 41 - 40 \cos 40 \]
   \[ = 10.358... \]
   \[ \Rightarrow AC = 3.2184... \]
   \[ = 3.22 \text{cm, correct to 3 s.f.} \]

5. Using the sine rule
   \[ \frac{PQ}{\sin 65} = \frac{RQ}{\sin 55} = \frac{7}{\sin(180° - 65° - 55°)} \]
   \[ \Rightarrow PQ = \frac{7 \sin 65}{\sin 60} = 7.32 \text{ cm, correct to 3 s.f.} \]
   \[ RQ = \frac{7 \sin 55}{\sin 60} = 6.62 \text{ cm, correct to 3 s.f.} \]
6 Draw the line BC and the line BA of 6 cm at an angle of 40°.
Then the line of 5 cm from A to BC can take two positions as shown.

\[ \sin C = \frac{\sin 40}{6} \]
\[ \Rightarrow \sin C = \frac{6 \times \sin 40}{5} = 0.7713... \]
\[ \Rightarrow C = 50.5° \text{, correct to 1 d.p.} \]
But also \( C = 180 - 50.5 = 129.5° \), correct to 1 d.p.

7 \[ \frac{\cos \theta + \sin \theta}{\cos \theta} = 1 + \frac{\sin \theta}{\cos \theta} \]
In a right-angled triangle:

\[ \sin \theta = \frac{a}{c}, \cos \theta = \frac{b}{c}, \tan \theta = \frac{a}{b} \]
\[ \frac{\sin \theta}{\cos \theta} = \frac{a/b}{c/b} = \frac{a}{c} = \tan \theta \]
\[ \Rightarrow \frac{\cos \theta + \sin \theta}{\cos \theta} = 1 + \frac{\sin \theta}{\cos \theta} = 1 + \tan \theta \]

8 \[ \sin \theta = 2 \cos \theta \Rightarrow \tan \theta = 2 \]
\[ \Rightarrow \theta = 63.4°, \text{ correct to 1 d.p.} \]
and also in 3rd quadrant, 243.4°

9 Angle of greatest slope = \( \angle CBE = \angle DAF \)
\[ \tan DAF = \frac{20}{50} \]
\[ \Rightarrow \angle DAF = 21.8° \]

10 Triangle CAE is a right angled.
AE is diagonal of base.
\[ AE = \sqrt{100^2 + 50^2} = 111.80 \]
Chapter 8 Trigonometric functions

8.1 Angles greater than 90°

Exam-style question (page 49)

(i)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
<th>330</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\theta + 60$</td>
<td>60</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>300</td>
<td>360</td>
<td>420</td>
<td>480</td>
<td>540</td>
<td>600</td>
<td>660</td>
<td>720</td>
<td>780</td>
</tr>
<tr>
<td>$y = \sin(2\theta + 60)$</td>
<td>0.87</td>
<td>0.87</td>
<td>0</td>
<td>-0.87</td>
<td>-0.87</td>
<td>0</td>
<td>0.87</td>
<td>0.87</td>
<td>0</td>
<td>-0.87</td>
<td>-0.87</td>
<td>0</td>
<td>0.87</td>
</tr>
</tbody>
</table>

(ii) The curves intersect at approximately $\theta = 117^\circ$

8.2 Sine and cosine rules

Exam-style question (page 52)

The triangle has two sides of 3.6 and 2.5 with the angle between of 35°.

Using the cosine rule to find the third side:

\[ p^2 = a^2 + b^2 - 2ab \cos \theta \]

\[ p^2 = 3.6^2 + 2.5^2 - 2 \times 3.6 \times 2.5 \cos 35^\circ \]

\[ = 4.465... \]

\[ \Rightarrow p = 2.113... \]

So Adam and Beth are 2.1 km apart, to 2 significant figures.
8.3 Identities and related equations

Exam-style question (page 53)

(i) \[ \text{LHS} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} = \text{RHS} \]

(ii) Since \( \sin \theta \cos \theta = \frac{1}{4} \) and \( \frac{\sin \theta}{\cos \theta} = \tan \theta \) and so \( \cos \theta = \frac{1}{\sin \theta \tan \theta} \), it follows that

\[ \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta \cos \theta} \]

\[ \Rightarrow \tan \theta + \frac{1}{\tan \theta} = 4 \text{ as required.} \]

(iii) Using \( t = \tan \theta \) gives

\[ t + \frac{1}{t} = 4 \]

\[ \Rightarrow t^2 + 1 = 4t \]

\[ \Rightarrow t^2 - 4t + 1 = 0 \]

\[ \Rightarrow t = \frac{4 \pm \sqrt{16 - 4}}{2} = 3.73 \text{ or } 0.268 \]

\[ \Rightarrow \theta = 75^\circ \text{ or } 15^\circ \]

Chapter 9 Applications of trigonometry

9.1 Applications of trigonometry

Exam-style questions (page 57)

The line VA meets the horizontal base at A and the angle that this line makes with the horizontal is VAO.
In the square base, ABCD,
\[ AC = \sqrt{6^2 + 6^2} = 8.485 \]
\[ \Rightarrow AO = \frac{1}{2} \times 8.49 = 4.243 \]
\[ \Rightarrow \text{Angle VAO} = \tan^{-1} \left( \frac{VO}{AO} \right) \]
\[ = \tan^{-1} \left( \frac{7}{4.243} \right) = 58.8^\circ \]

Review questions (Chapters 8–9)
(pages 58–59)

1. \[ \sin 2x = 0.5 \]
   \[ \Rightarrow 2x = 30^\circ, 150^\circ \]
   \[ \Rightarrow x = 15^\circ, 75^\circ \]
   Also \( x = 15^\circ + 180^\circ = 195^\circ \)
   and \( x = 75^\circ + 180^\circ = 255^\circ \)

2. \( \tan x \) is positive in the first and third quadrants.
   \( \cos x \) is negative in the second and third quadrants.
   Therefore the angle required is in the third quadrant.
   \[ \tan x = 0.75 \]
   \[ \Rightarrow x = 36.9^\circ, 216.9^\circ \]
   So the required angle is \( 216.9^\circ \)

3. \[ \sin x = -2\cos x \]
   \[ \Rightarrow \tan x = -2 \]
   \[ \Rightarrow x = 116.6^\circ, 296.6^\circ \]

4. (i) \[ \text{Area} = \frac{1}{2} \times 6 \times 8 = 24 \]
   So area = 24 cm\(^2\)

   (ii)
   \[ \text{Area} = \frac{1}{2} \times 6 \times h = 20 \]
   \[ \Rightarrow h = \frac{40}{6} = \frac{20}{3} \]
   Also \( h = 8 \sin B \)
   \[ \Rightarrow \sin B = \frac{h}{8} = \frac{5}{6} \]
   \[ \Rightarrow B = 56.4^\circ \]
   but also \( B = 123.6^\circ \)
5 \(\frac{b^2 = a^2 + c^2 - 2ac \cos B}{\Rightarrow b^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos 50} \Rightarrow b = 9.47 \text{ m}\)

**Two sides and the included angle means the cosine formula.**

**Common mistake:** Note that \(244 - 240 \cos 50\) is not \(4 \cos 50\).

6 \(\sin^2 \theta = 12 - 13 \cos \theta \Rightarrow 1 - \cos^2 \theta = 12 - 13 \cos \theta \Rightarrow \cos^2 \theta - 13 \cos \theta + 11 = 0 \Rightarrow \cos \theta = \frac{13 \pm \sqrt{13^2 - 4 \times 11}}{2} = \frac{13 \pm \sqrt{125}}{2} = \frac{13 \pm 11.2}{2} \Rightarrow \cos \theta = 2.6 \text{ and } 0.9 = 0.9 \Rightarrow \cos \theta = 0.9 \Rightarrow \theta = 25.8^\circ \text{ or } 360^\circ - 25.8^\circ = 334.2^\circ \) [Use the identity \(\sin^2 \theta + \cos^2 \theta = 1\).]

**Solve as a quadratic in \(\cos \theta\).**

**When you reject one possible answer you should say why.**

**+ve \(\cos \theta\) means \(\theta\) is in the first and fourth quadrants.**

7 In the triangle \(a^2 + b^2 = c^2\)

Divide by \(a^2 \Rightarrow 1 \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2\)

Since \(\frac{b}{a} = \tan \theta\) and \(\frac{a}{c} = \cos \theta\)

\(1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}\)

**This is a `show that` question which means that you must show all your working.**

8 (i) In triangle \(ACV\),

\(\frac{AC}{10} = \tan 40 \Rightarrow AC = 10 \tan 40 = 8.39\)

(ii) In triangle \(ACB\),

Angle \(ACB = 180^\circ - 50^\circ - 60^\circ = 70^\circ\)

Using sin rule

\(\frac{AB}{\sin ACB} = \frac{AC}{\sin CBA}\)

\(\Rightarrow AB = \frac{8.39}{\sin 60} \Rightarrow AB = \frac{8.39 \times \sin 70}{\sin 60} = 9.10\)

9 (i) \(\frac{100}{BE} = \sin 30 \Rightarrow BE = \frac{100}{\sin 30} = 200\)

(ii) \(\frac{AE}{BE} = \sin 30 \Rightarrow AE = \frac{100}{\sin 30} = 200\)

**BEC is a right-angled triangle; it would be helpful if you drew it.**

**ABE is a right-angled triangle; it would be helpful if you drew it.**

**Use Pythagoras since you know the lengths of two sides, one from the question and the other is the answer to (i). That gives the length \(AE\) for the triangle \(ACE\).**

Use Pythagoras since you know the lengths of two sides, one from the question and the other is the answer to (i). That gives the length \(AE\) for the triangle \(ACE\).
(iii) Area of ABE = \( \frac{1}{2} \times 500 \times \text{your value for BE} \)
\[ = 50000 \]
Area of ABE = \( \frac{1}{2} \times BG \times \text{your value for AE} \)
\[ \Rightarrow BG = \frac{2 \times 50000}{538.5} = 185.7...... = 186 \]

This must come from areas.
Section 4 Selections

Target your revision (Chapters 10–11) (page 60)

1 Fill in the number of boys brought by car, the number of girls who came by bus, the number of girls who walked and the total number who were brought by car.

<table>
<thead>
<tr>
<th></th>
<th>Walked</th>
<th>Brought by car</th>
<th>Came by bus</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>5</td>
<td>3</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Girls</td>
<td>4</td>
<td></td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>8</td>
<td>13</td>
<td>30</td>
</tr>
</tbody>
</table>

Then the last two can be filled in.

<table>
<thead>
<tr>
<th></th>
<th>Walked</th>
<th>Brought by car</th>
<th>Came by bus</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>5</td>
<td>3</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Girls</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>8</td>
<td>13</td>
<td>30</td>
</tr>
</tbody>
</table>

2 The information of the question gives the following numbers in each part of the Venn diagram.

The separate parts sum to 30.
So the number outside, with no brothers or sisters, is 40 − 30 = 10

3 This is the tree diagram for Ahmed.

Probability that Ahmed wins both games = 0.28

4 \[ \frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = \frac{7 \times 6 \times 5}{1} = 210 \]
5 (i) The cards may be laid down on the table to create a 5-digit number in $5! = 120$ ways.

(ii) Two of the cards are even. So there are 2 ways to place the card on the right.

The other 4 may be placed on the table in $4! = 24$ ways.

So 48 of the numbers are even.

6 Order does not matter so

$$\frac{30!}{3!} \times \frac{30 \times 29 \times 28}{3 \times 2 \times 1} = 5 \times 29 \times 28 = 4060$$

7 (i) $P(1\text{st ball is red}) = \frac{4}{8} = \frac{1}{2}$

If the first ball is replaced then the situation is the same.

So $P(2\text{nd ball is red}) = \frac{4}{8} = \frac{1}{2}$

$\Rightarrow P(\text{both balls are red}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(ii) $P(1\text{st ball is red}) = \frac{4}{8} = \frac{1}{2}$

If the first ball is not replaced then the situation is not the same.

So $P(2\text{nd ball is red given that the first ball is red}) = \frac{3}{7}$

$\Rightarrow P(\text{both balls are red}) = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

8 5th row of Pascal's triangle is 1 5 10 10 5 1

So expansion is

$$(2-3x)^5 = 2^5 + 5 \cdot 2^4 (-3x)^1 + 10 \cdot 2^3 (-3x)^2 + 10 \cdot 2^2 (-3x)^3 + 5 \cdot 2 (-3x)^4 + (-3x)^5$$

$= 32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5$

9

$$\left(1 + \frac{1}{2}x\right)^{10} = 1 + 10 \left(\frac{1}{2}x\right) + 45 \left(\frac{1}{2}x\right)^2 + 120 \left(\frac{1}{2}x\right)^3 + 210 \left(\frac{1}{2}x\right)^4 + ...$$

$$= 1 + 5x + \frac{45}{4} x^2 + 15x^3 + \frac{105}{8} x^4 + ...$$

10

$$\left(2x - \frac{3}{x}\right)^6 = (2x)^6 + \binom{6}{1} (2x)^5 \left(-\frac{3}{x}\right) + \binom{6}{2} (2x)^4 \left(-\frac{3}{x}\right)^2 + \binom{6}{3} (2x)^3 \left(-\frac{3}{x}\right)^3 + ...$$

The constant term is the one where the powers are the same, i.e. the 4th term.

$$\binom{6}{3} (2x)^3 \left(-\frac{3}{x}\right)^3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 2^3 (-3)^3 = -20 \times 8 \times 27 = -4320$$

11 (i) $P(\text{a component is faulty}) = \frac{1}{10}$

$P(\text{a component is good}) = \frac{9}{10}$

$P(\text{all 10 components in a box are good}) = \left(\frac{9}{10}\right)^{10} = 0.3486...$

(ii) $P(\text{all 10 are good}) + P(\text{1 is faulty}) + P(\text{2 or more are faulty}) = 1$

$\Rightarrow P(\text{2 or more are faulty}) = 1 - \left(\frac{9}{10}\right)^{10} - 10 \left(\frac{9}{10}\right)^9 \left(\frac{1}{10}\right)$

$= 1 - 0.3486... - 0.3874... = 0.2639...$

$= 0.264$ (3 s.f.)
Chapter 10 Permutations and combinations

10.1 Definitions and probability
Exam-style question (page 64)

(i) Numbers in each region are 23, 5, 5, 2, 4 totalling 39
45 − 39 = 6
So, 6 people did not buy any of these three items.

(ii) There are 3 circles in the Venn diagram.
No one has both biscuits and sandwiches so those two circles do not overlap.
In an examination the three circles overlapping with each other would be accepted providing the number inside the redundant overlap is 0.
Start by putting 2 and 5 into the respective overlaps. Then calculate the numbers in each of the non-overlapping regions.

They sum to 39 leaving 6 others.

10.2 Factorials, permutations and combinations
Exam-style question (page 66)

(i) 6! = 720

(ii) There are now only 5 seats for the remaining 5 friends.
So, there are 5! = 120 ways.

(iii) Initially, consider Jane and Julian as one person taking 2 seats.
So there are now only 4 seats for 4 'people' meaning 4! = 24 ways.
But for each one of those ways there are 2 ways for Jane and Julian to sit.
So the total number of ways is 2 × 4! = 48
So there are 48 different arrangements.

Chapter 11 The binomial distribution

11.1 The binomial expansion
Exam-style questions (page 68)

(i) \[(1-x)^3 = 1 + \binom{3}{1}(-x) + \binom{3}{2}(-x)^2 + \binom{3}{3}(-x)^3\]
\[= 1 - 12x + \frac{12 \times 11}{1 \times 2} \cdot (-x)^3 - \frac{12 \times 11 \times 10}{1 \times 2 \times 3} \cdot x^3\]
\[= 1 - 12x + 66x^2 - 220x^3\]

(ii) The expansion is as follows.
\[\left(2x - \frac{3}{x}\right)^8 = \left(2x\right)^8 + 8\left(2x\right)^7\left(-\frac{3}{x}\right) + ... + 8\left(2x\right)\left(-\frac{3}{x}\right)^7 + \left(-\frac{3}{x}\right)^8\]
There is a term in the middle where the powers of \(x\) cancel out.
This term is \(\binom{8}{4} \cdot (2x)^4 \left(-\frac{3}{x}\right)^4\)
\[= \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \cdot 2^4 \cdot (-3)^4 = 70 \times 16 \times 81 = 90720\]

The powers add to 8 so both must be 4.
Remember to include the 2 and the −3.
The coefficient can either be obtained from Pascal’s triangle from the row starting 1, 8, ...
or from the formula
\[\frac{8!}{(8-4)!} \cdot \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70\]
11.2 The binomial distribution

Exam-style question (page 70)

(a) \( P(\text{red}) = \frac{7}{10}, P(\text{green}) = \frac{3}{10} \)

(i) \( P(\text{all 5 red}) = \left( \frac{7}{10} \right)^5 = 0.168 \)

(ii) \( P(2 \text{ red}) = \left( \frac{5}{2} \right) \left( \frac{7}{10} \right)^2 \left( \frac{3}{10} \right)^3 = 0.132 \)

(b) There are two outcomes – green or red (equivalent to ‘success’ and ‘failure’).

The probability of picking a red marble remains constant.

Picking a marble is independent of the previous selection.

Review questions (Chapters 10–11) (page 71)

1 (i)

\[ \begin{array}{c|cccccc}
\text{1st die} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 3 & 4 & 5 & 6 & 7 & 8 \\
2 & 5 & 6 & 7 & 8 & 9 & 10 \\
3 & 7 & 8 & 9 & 10 & 11 & 12 \\
4 & 9 & 10 & 11 & 12 & 13 & 14 \\
5 & 11 & 12 & 13 & 14 & 15 & 16 \\
6 & 13 & 14 & 15 & 16 & 17 & 18 \\
\end{array} \]

(ii) \( \frac{3}{36} = \frac{1}{12} \)

2 (i)

The ‘rule’ in this question is to double the number on the first die and add to the number on the second die.

(ii) \( \frac{5 \times 5}{10 \times 9} = \frac{25}{90} = \frac{5}{18} \)

Follow the lines representing the outcomes and multiply the probabilities.

Simplify your answer.

In an examination, a choice from a ‘large number’ implies that the probability of success remains constant. So, in this case, whatever the colour of the first marble chosen, the proportion of colours remains 7:3.

If there were just 10 marbles, 7 red and 3 green, then the probability of choosing a red marble second would depend on what the first choice was. The binomial distribution in this case would not be appropriate.

Five marbles are drawn. So exactly two reds means that the other three are green.

In the table above 12 appears 3 times.

There are 36 entries.

Simplify your fraction.

The tree should have the outcomes labelling the branches and the associated probabilities.

Note that in this situation the probabilities are dependent as, after the first draw there are only nine balls left from which to choose and the mix of colours depend on the colour of the ball drawn first.

Follow the lines representing the outcomes and multiply the probabilities.

Simplify your answer.
Red, blue or blue, red both of which have the same probability.

This is a combination as order does not matter.

The chairman can be chosen in 10 ways.

Now there are only 9 people from whom to choose the remaining 4.

There is no choice about the middle seat. There are now 6 people left to fill 6 seats. This is a permutation.

There are two more places now fixed and 4 people to fill 4 places. But Pedro and Quentin can sit at either end, doubling the permutation.

Write out all the terms taking care to include the −ve sign. Note that in the final expansion the terms are alternatively positive and negative.

Equate coefficients

Substitute for \( a \) and solve for \( n \).

Substitute back to find the value of \( a \).
Review questions (Chapters 10–11)

(iii) \[ \left( 1 - \frac{4}{x} + \frac{4}{x^2} \right) \left( 1 + 4x + 7x^2 + 7x^3 \right) \]
\[ = 1 \frac{4}{x} \times 4x + \frac{4}{x^2} \times 7x^2 \]
\[ = 1 - 16 + 28 = 13 \]

8 (i) \[ (p + q)^3 = \ldots + \left( \frac{4}{2} \right) p^2 q + \ldots \]
where \[ p = q = \frac{1}{2} \]
\[ \Rightarrow 6 \times \left( \frac{1}{2} \right)^4 = 6 \frac{16}{6} = \frac{3}{8} \]

(ii) \[ P(\text{at least one tail}) = 1 - P(0 \text{ tails}) \]
\[ = 1 - \left( \frac{1}{2} \right)^4 = \frac{15}{16} \]

9 (i) \[ P(\text{male}) = \frac{9}{20}, P(\text{female}) = \frac{11}{20} \]
\[ P(\text{all are males}) = \left( \frac{9}{20} \right)^8 = 0.00168 \]

(ii) \[ \left( \frac{8}{5} \right) \left( \frac{9}{20} \right) \left( \frac{11}{20} \right)^5 \]
\[ = 56 \times 0.45^3 \times 0.55^5 \]
\[ = 0.257 \]

(iii) \[ P(\text{part-time}) = \frac{1}{3} \times \frac{9}{20} + \frac{1}{2} \times \frac{11}{20} \]
\[ = \frac{3}{20} + \frac{11}{40} = \frac{17}{40} \]

\[ P(\text{at least 2 part-time employees}) \]
\[ = 1 - P(0 \text{ part-time}) - P(1 \text{ part-time}) \]
\[ = 1 - \left( \frac{23}{40} \right)^8 - 8 \left( \frac{23}{40} \right)^7 \left( \frac{17}{40} \right) \]
\[ = 1 - 0.0119 - 0.0706 \]
\[ = 0.917 \]

There are three terms where the powers of \( x \) cancel.

This is the middle term of the binomial expansion.

There are two outcomes, at least one tail and no tails. Their probabilities sum to 1.

This is a situation where the word 'large' implies that the ratio of male : female does not change when you pick out one employee. So the probability of choosing a male remains the same for all choices.

In this situation you will be expected to give a 3 significant figure decimal answer.

Choose the appropriate term in the binomial distribution.

First work out the probability of picking a part-time employee and so of not picking a part-time employee.

First work out the probability of picking a part-time employee and so of not picking a part-time employee.

The three outcomes (0 part-time or one part-time or at least two part-time) sum to 1.
Target your revision (Chapters 12–13)
(pages 72–73)

1

2 (i) \( m = 20 \times 1.7^{-0.8t} \)
\( t = 0 \Rightarrow m = 20 \)

(ii) When \( t = 6 \), \( m = 20 \times 1.7^{-4.8} = 1.57 \), correct to 3 s.f.

3 \( 3 \log 2 - \frac{1}{2} \log 14 \)
\( = \log 8 - \log \sqrt{14} \)
\( = \log \left( \frac{8}{\sqrt{14}} \right) \)
\( = \log \left( \frac{4\sqrt{14}}{7} \right) \)

4 (i) \( y = k a^x \Rightarrow \log y = \log k + x \log a \)
So plotting \( \log y \) against \( x \) will give a straight line with intercept \( \log k \) and gradient \( \log a \).
Writing the logarithms to 3 decimal places gives the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.375</td>
<td>0.650</td>
<td>1.125</td>
<td>1.950</td>
<td>3.375</td>
</tr>
<tr>
<td>( \log y )</td>
<td>-0.426</td>
<td>-0.187</td>
<td>0.051</td>
<td>0.290</td>
<td>0.528</td>
</tr>
</tbody>
</table>

The points do lie on a line, so the model is valid.
(iii) Intercept is \(-0.9 = \log k \Rightarrow k = 0.1\)

Gradient is \(\frac{0.53 - (-0.43)}{3 - 1} = 0.48 = \log a \Rightarrow a = 3\)

5 \(y = kx^n \Rightarrow \log y = \log k + n \log x\)
So plotting \(\log y\) against \(\log x\) should give a straight line with gradient \(n\) and intercept \(\log k\).

To show that the data are well modelled by the relationship a graph is essential.

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0.1</td>
<td>0.5</td>
<td>1.6</td>
<td>3.9</td>
<td>8.1</td>
</tr>
<tr>
<td>(\log x)</td>
<td>0</td>
<td>0.176</td>
<td>0.301</td>
<td>0.398</td>
<td>0.477</td>
</tr>
<tr>
<td>(\log y)</td>
<td>-1</td>
<td>-0.301</td>
<td>0.204</td>
<td>0.591</td>
<td>0.908</td>
</tr>
</tbody>
</table>

The points do lie on a line, so the model is valid

Intercept is \(-1 = \log k \Rightarrow k = 0.1\)

Gradient is \(\frac{0.91 - (-1)}{0.48 - 0} = 3.979 = 4 = n\)

6 \(2^x = 7\)
\(\Rightarrow \log 2^x = \log 7\)
\(\Rightarrow x \log 2 = \log 7\)
\(\Rightarrow x = \frac{\log 7}{\log 2} = 2.81, \text{ correct to 3 s.f.}\)
7 \[ 2^{x+1} = 3^{2-3x} \]
\[ \Rightarrow (x+1)\log 2 = (2 - 3x)\log 3 \]
\[ \Rightarrow x(\log 2 + 3\log 3) = 2\log 3 - \log 2 \]
\[ \Rightarrow x \log 54 = \log 4.5 \]
\[ \Rightarrow x = \frac{\log 4.5}{\log 54} = 0.377, \text{ correct to 3 s.f.} \]

8 \[ f(x) = x^3 + x - 3 \]
\[ f(0) = -3, \ f(1) = -1, \ f(2) = 7 \text{ so the root lies in interval [1, 2].} \]

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1.1</td>
<td>-0.569</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.072</td>
</tr>
<tr>
<td>1.3</td>
<td>0.497</td>
</tr>
</tbody>
</table>

so the root lies in interval [1.2, 1.3].

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>-0.072</td>
</tr>
<tr>
<td>1.21</td>
<td>-0.01844</td>
</tr>
<tr>
<td>1.22</td>
<td>0.035848</td>
</tr>
</tbody>
</table>

so the root lies in interval [1.21, 1.22].
\[ f(1.215) = 0.0086 > 0 \text{ so the root is 1.21 correct to 2 decimal places.} \]

9 \[ x^3 - 2x - 2 = 0 \]
\[ f(1) = -3, \ f(2) = 2 \text{ so root is in interval [1, 2] with mid-interval 1.5.} \]
Try \[ x = 1.5, \ f(1.5) = -1.625 \text{ so root is in interval [1.5, 2] with mid-interval 1.75.} \]
Try \[ x = 1.75, \ f(1.75) = -0.141 \text{ so root is in interval [1.75, 2] with mid-interval 1.875.} \]
Try \[ x = 1.875, \ f(1.875) = 0.842 \text{ so root is in interval [1.75, 1.875] with mid-interval 1.8125.} \]
Try \[ x = 1.8125, \ f(1.8125) = 0.329 \text{ so root is in interval [1.75, 1.8125] with mid-interval 1.78125.} \]
Try \[ x = 1.78125, \ f(1.78125) = 0.0891 \text{ so root is in interval [1.75, 1.78125] with mid-interval 1.765625.} \]
Try \[ x = 1.765625, \ f(1.765625) = -0.0270 \text{ so root is in interval [1.765625, 1.78125] with mid-interval 1.7734375.} \]
Try \[ x = 1.7734375, \ f(1.7734375) = 0.0307 \text{ so root is in interval [1.765625, 1.7734375].} \]
The upper and lower bounds both round to 1.77.
So, correct to 2 decimal places, the root is 1.77.

10 (i) \[ f(1) = -3, \ f(2) = 3 \]

(ii) \[ x^3 - x - 3 = 0 \]
\[ \Rightarrow x^3 = x + 3 \]
\[ \Rightarrow x = \sqrt[3]{x + 3} \]
(iii) | $x_r$  | $x_{r+1}$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.587401</td>
</tr>
<tr>
<td>2</td>
<td>1.661584</td>
</tr>
<tr>
<td>3</td>
<td>1.670492</td>
</tr>
<tr>
<td>4</td>
<td>1.671556</td>
</tr>
<tr>
<td>5</td>
<td>1.671683</td>
</tr>
</tbody>
</table>

It appears that the root is 1.6717, correct to 4 d.p.
To be sure you should calculate the value of the function either
side of this value to confirm a change of sign.
For example, $f(1.67165) = -0.0004 < 0$ and $f(1.67175) = 0.0004 > 0$.
So the root is 1.6717, correct to 4 d.p.

11 (i) Gradient of chord $\frac{2.60 - 0.42}{3-1} = \frac{2.18}{2} = 1.09$

Because $A$ and $C$ are equidistant either side of $B$, the chord will be
approximately parallel to the tangent at $B$.
So the gradient of the tangent can be estimated to be 1.1.

12 (i) Gradient of chord $\frac{2.60 - 1.04}{3-2} = \frac{1.56}{1} = 1.56$
This is greater than the tangent at $B$

Gradient of chord $\frac{1.04 - 0.42}{2-1} = \frac{0.62}{1} = 0.62$
This is less than the gradient of the tangent at $B$.

(ii) This gives $0.62 < \text{gradient of tangent at } B < 1.56$
A better estimate than either of these is the mean of them, that is, 1.09
13 The points on the curve are (0, 8), (1, 4), (2, 2) and (3, 1).
   (i) Three rectangles under the curve have heights 4, 2 and 1.
       So area = 7 square units.
   (ii) Three rectangles above the curve have heights 8, 4 and 2
       So area = 14 square units.
   (iii) A better estimate is the mean of these values. i.e. 10.5 square units

14 The total area of the 3 trapezia

\[
\frac{1}{2}(8+4) \times 1 + \frac{1}{2}(4+2) \times 1 + \frac{1}{2}(2+1) \times 1 \\
= 6 + 3 + 1.5 \\
= 10.5 \text{ square units}
\]

Chapter 12 Exponentials and logarithms

12.1 Exponential functions
Exam-style question (page 76)
Take \( V = ka^t \)
where \( V \) is measured in thousands of pounds and \( t \) is measured in years.
The initial condition is \( V = 16 \) when \( t = 0 \).
\[ \Rightarrow k = 16 \]
The second condition is that \( V = 12 \) when \( t = 1 \).
\[ \Rightarrow 12 = 16a^{-1} \]
\[ \Rightarrow a^{-1} = \frac{3}{4} \]
\[ \Rightarrow a = \frac{4}{3} \]
\[ \Rightarrow V = 16 \left( \frac{4}{3} \right)^t \]

When \( t = 5 \), \( V = 16 \left( \frac{4}{3} \right)^5 = 3.797 \) (to 3 d.p.)
i.e. the value of the car is approximately £3797 or a little less than £3800.

12.2 Logarithms
Exam-style question (page 78)
(i) \( 2 \log_{10} x - \log_{10} 6 \)
\[ = \log_{10} x^2 - \log_{10} 6 = \log_{10} \left( \frac{x^2}{6} \right) \]
(ii) \( 2 \log_{10} x - \log_{10} 6 = 1.2 \)
\[ \Rightarrow \log_{10} \left( \frac{x^2}{6} \right) = 1.2 \]
\[ \Rightarrow \frac{x^2}{6} = 10^{1.2} = 15.849 \]
\[ \Rightarrow x^2 = 95.09 \]
\[ \Rightarrow x = 9.75 \]
12.3 Reduction to linear form
Exam-style question (page 81)

(i) \( y = ka^x \)

\[ \Rightarrow \log y = \log k + \log a^x = \log k + x \log a \]

If the relationship is correct then plotting \( \log y \) against \( x \) will give a straight line with \( \log k \) the intercept on the \( y \)-axis and \( \log a \) the gradient.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>120</td>
<td>144</td>
<td>172</td>
<td>208</td>
<td>248</td>
<td>300</td>
</tr>
<tr>
<td>( \log y )</td>
<td>2.08</td>
<td>2.16</td>
<td>2.24</td>
<td>2.32</td>
<td>2.39</td>
<td>2.48</td>
</tr>
</tbody>
</table>

A straight line results, so the data can be assumed to satisfy the relationship \( y = ka^x \).

(ii) Intercept:

The line passes through \((0, 2)\)

\[ \Rightarrow \log k = 2 \]

\[ \Rightarrow k = 10^2 \]

\[ \Rightarrow k = 100 \]

Gradient:

\[ \frac{2.48 - 2.08}{5} = 0.08 \]

\[ \Rightarrow \log a = 0.08 \]

\[ \Rightarrow a = 1.2 \]

So the relationship is \( y = 2 \times 1.2^x \).

(iii) Once the number of customers reaches the number of people in the town who buy meat, it can grow no further.

12.4 Solving exponential equations
Exam-style question (page 82)

(i) \( T - 25 = 75 \times a^{\frac{t}{4}} \)

\[ \Rightarrow \log(T - 25) = \log 75 - \frac{t}{4} \log a \]

So plotting \( \log(T - 25) \) against \( t \) will give a straight line with intercept \( \log 75 \) and gradient \(-\frac{\log a}{4}\). Note that the left-hand side is \( T - 25 \) and not just \( T \) so that when \( t = 0, T = 100 \). You would be unable to evaluate the right-hand side if you tried taking \( \log \) of \( 25 + 75 \times a^{\frac{t}{4}} \).
(ii) The values required, using base 10.

<table>
<thead>
<tr>
<th>t</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>68.3</td>
<td>50</td>
<td>39.4</td>
<td>33.3</td>
<td>29.8</td>
</tr>
<tr>
<td>T - 25</td>
<td>43.3</td>
<td>25</td>
<td>14.4</td>
<td>8.3</td>
<td>4.8</td>
</tr>
<tr>
<td>log(T - 25)</td>
<td>1.64</td>
<td>1.40</td>
<td>1.16</td>
<td>0.92</td>
<td>0.68</td>
</tr>
</tbody>
</table>

The points are plotted as shown below. They do indeed lie on a straight line.

In order to find the gradient and intercept you should draw the line of best fit.

From the graph the intercept is 1.875 and the gradient is −0.12
Intercept \(10^{1.875} = 75\) i.e. \(T - 25 = 75\) as required
Gradient
\[
\frac{\log a}{4} = -0.12
\]
\[\Rightarrow \log a = 0.48\]
\[\Rightarrow a = 3.02\]
So \(a = 3\)

(iii) As \(t\) increases, \(75a^{-\frac{t}{4}}\) becomes closer to zero but never becomes negative.
Therefore, when \(t\) is large, \(T - 25 = 0\) and \(T = 25\) so temperature approaches 25 °C

Chapter 13 Numerical methods

13.1 Sign change methods for solving an equation

Exam-style question (page 85)

(i) \(f(1) = -2 < 0, f(2) = 6 > 0\)
There is a root in the interval \([1, 2]\)
(ii) \( f(1.5) = 0.875 > 0 \)

The root is in the interval \([1, 1.5]\)

\( f(1.25) = -0.797 < 0 \)

The root is in the interval \([1.25, 1.5]\)

\( f(1.375) = -0.02539 < 0 \)

The root is in the interval \([1.375, 1.5]\)

\( f(1.4375) = 0.408 > 0 \)

The root is in the interval \([1.375, 1.4375]\)

This shows that to 2 significant figures, the root is \(x = 1.4\).

### 13.2 Iterative methods

**Exam-style question (page 88)**

(i) \( x = 10^{-x^2} \Rightarrow \log x = x - 2 \)

\( \Rightarrow x = \log x + 2 \)

(ii) 

![Graph](image)

(iii) 

![Graph](image)

(iv) 

<table>
<thead>
<tr>
<th>(x_n)</th>
<th>(x_{n+1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2.30103</td>
</tr>
<tr>
<td>2.30103</td>
<td>2.361922</td>
</tr>
<tr>
<td>2.361922</td>
<td>2.373266</td>
</tr>
<tr>
<td>2.373266</td>
<td>2.375346</td>
</tr>
<tr>
<td>2.375346</td>
<td>2.375727</td>
</tr>
</tbody>
</table>

So the root, correct to 3 significant figures, is \(x = 2.38\).
13.3 Gradients of tangents
Exam-style question (page 89)

(i) and (ii)

![Graph showing a tangent at a point]

(iii) Using (2, 2.25) and (4, 20.25)

\[\text{Gradient} = \frac{20.25 - 2.25}{4 - 2} = 9\]

(iv) Accuracy can be improved by taking points closer to the point where \(x = 3\). In this case perhaps \(x = 2.5\) and \(x = 3.5\).

13.4 The area under a curve
Exam-style question (page 91)

(i)

![Graph showing a curve and trapeziums]

(ii) \(A = \frac{1}{2} h \left( y_0 + 2(y_1 + y_2) + y_3 \right)\)

\[= \frac{1}{2} \left( 13 + 7 + 2(17 + 13) \right)\]

\[= 40\]

The first 2 trapezia are under the curve but the third is just over. The estimate will be an underestimate.
Review questions (Chapters 12–13) (page 92)

1

\[ y = ka^x \]

For A:

\[ 4.5 = ka^2 \]

For B:

\[ 13.5 = ka^3 \]

Divide:

\[ \frac{13.5}{4.5} = \frac{ka^3}{ka^2} \]

\[ a = 3 \]

Substitute:

\[ 4.5 = 9k \]

\[ k = 0.5 \]

2 Let the sum of money invested be \( P \).

Then at the end of the \( n^{th} \) year the sum is \( P \times 1.05^n = 2P \)

\[ n \log 1.05 = \log 2 \]

\[ n = \frac{\log 1.05}{\log 2} = 14.206 \ldots \]

So the sum is doubled by the end of the 15th year.

3 \( \log_{10} x - \log_{10} 2 = 2 \)

\[ \Rightarrow \log_{10} \frac{x}{2} = 2 \]

\[ \Rightarrow \frac{x}{2} = 10^2 = 100 \]

\[ \Rightarrow x = 200 \]

4 \[ 3\log_2 4 - 5\log_2 8 \]

\[ = 3\log_2 2^2 - 5\log_2 2^3 \]

\[ = 6\log_2 2 - 15\log_2 2 \]

\[ = 6 - 15 = -9 \]

5 (i) \[ y = ka^x \Rightarrow \log y = \log(ka^x) = \log k + \log a^x \]

\[ \Rightarrow \log y = \log k + x \log a \]

This is the form \( Y = mX + c \) where \( Y = \log y \) and \( X = x \).

The gradient of the line is \( \log a \) and the intercept on the \( y \)-axis is \( \log k \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.7</td>
<td>2.5</td>
<td>3.8</td>
<td>4.7</td>
<td>8.5</td>
</tr>
<tr>
<td>( \log y )</td>
<td>0.23</td>
<td>0.40</td>
<td>0.58</td>
<td>0.67</td>
<td>0.93</td>
</tr>
</tbody>
</table>

(iii) \( \log y \)

(iii) The point (5, 4.7) does not fit the model (\( \log y \) should be 0.76 not 0.67).
log y = log k + x log a

The gradient is approximately 0.18 and the intercept on the y-axis is approximately −0.14

i.e. \( \log a = 0.18 \Rightarrow a = 1.5 \)

\( \log k = -0.14 \Rightarrow k = 0.72 \)

6 (i) \( y = kx^n \)

Take logs: \( \log y = \log k + n \log x \)

So plot \( \log y \) against \( \log x \): the intercept will be \( \log k \) and the gradient will be \( n \).

(ii) | x  | 1  | 2  | 3  | 4  | 5  | 6  |
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.5</td>
<td>4</td>
<td>13.5</td>
<td>32</td>
<td>62.5</td>
<td>108</td>
</tr>
<tr>
<td>\log x</td>
<td>0</td>
<td>0.30</td>
<td>0.48</td>
<td>0.60</td>
<td>0.70</td>
<td>0.78</td>
</tr>
<tr>
<td>\log y</td>
<td>-0.30</td>
<td>0.60</td>
<td>1.13</td>
<td>1.51</td>
<td>1.80</td>
<td>2.03</td>
</tr>
</tbody>
</table>

(iii) Intercept on y axis is \( -0.3 = \log k \)

\( \Rightarrow k = 10^{-0.3} = 0.5 \)

Gradient \( = \frac{2.03 - (-0.3)}{0.78 - 0} = n = 3 \)
7 \(3^{x+1} = 2^{2x-1}\)
Take logs:
\[(x+1)\log 3 = (2x-1)\log 2\]
\[\Rightarrow x(\log 3 - 2\log 2) = -\log 2 - \log 3\]
\[\Rightarrow x\log \frac{3}{4} = -\log 6\]
\[\Rightarrow x = \frac{-\log 6}{\log \frac{3}{4}} = 6.23\]

Any base will do but for numerical work it is easiest to use base 10.

8 \(5000 \times 1.03^n = 7000\)
\[\Rightarrow 1.03^n = \frac{7000}{5000} = 1.4\]
\[\Rightarrow \log 1.03^n = n \log 1.03 = \log 1.4\]
\[\Rightarrow n = \frac{\log 1.4}{\log 1.03} = 11.38\]

Each year the sum is increased by 3% so multiply by 1.03 for each year.

After 11 years the sum is not quite £7000.

So the sum exceeds £7000 by the end of the 12th year.
Section 6 Calculus

Target your revision (Chapters 14–16) (pages 93–94)

1. \[ y = 2x^2 + \frac{1}{2}x \]
   \[ \Rightarrow \frac{dy}{dx} = 2 \times 2x + \frac{1}{2} = 4x + \frac{1}{2} \]

2. \[ y = x^3 - 2x^2 + 3x + 1 \]
   \[ \Rightarrow \frac{dy}{dx} = 3x^2 - 4x + 3 \]

   When \( x = 1 \), \( \frac{dy}{dx} = 3 - 4 + 3 = 2 \)

3. \[ y = x^3 + x^2 - 4x + 3 \]
   \[ \Rightarrow \frac{dy}{dx} = 3x^2 + 2x - 4 \]

   When \( x = 2 \), \( \frac{dy}{dx} = 3 \times 4 + 2 \times 2 - 4 = 12 \)
   \[ \Rightarrow \text{Gradient of tangent} = 12 \]
   \[ \Rightarrow \text{Equation of tangent is} \quad y - 7 = 12(x - 2) \]
   \[ \Rightarrow \quad y = 12x - 17 \]

4. \[ \frac{dy}{dx} = 3x^2 + 2x - 4 \]

   When \( x = 2 \), \( \frac{dy}{dx} = 3 \times 4 + 2 \times 2 - 4 = 12 \)
   \[ \Rightarrow \text{Gradient of tangent} = 12 \]
   \[ \Rightarrow \text{Gradient of normal} = -\frac{1}{12} \]
   \[ \Rightarrow \text{Equation of normal is} \quad y - 7 = -\frac{1}{12}(x - 2) \]
   \[ \Rightarrow \quad 12y + x = 86 \]

5. \[ y = x^3 + 3x^2 - 9x - 10 \Rightarrow \frac{dy}{dx} = 3x^2 + 6x - 9 \]

   \[ = 0 \] when \( 3x^2 + 6x - 9 = 0 \Rightarrow x^2 + 2x - 3 = 0 \)
   \[ \Rightarrow (x + 3)(x - 1) = 0 \]
   \[ \Rightarrow x = -3 \text{ giving } y = 17 \]
   \[ \text{and } x = 1 \text{ giving } y = -15 \]

   Coordinates of intersection are \((-3, 17)\) and \((1, -15)\).

   \[ \frac{d^2y}{dx^2} = 6x + 6 \]

   when \( x = -3 \), \( \frac{d^2y}{dx^2} = -12 < 0 \) so \((-3, 17)\) is a maximum

   when \( x = 1 \), \( \frac{d^2y}{dx^2} = 12 > 0 \) so \((1, -15)\) is a minimum
6 \int (3x^2 - 3x + 1) \, dx
= x^3 - \frac{3x^2}{2} + x + c

7 \frac{dy}{dx} = 1 + 2x - 4x^3
\Rightarrow y = x + x^2 - x^4 + c
Substitute (2, 5)
\Rightarrow 5 = 2 + 4 - 16 + c
\Rightarrow c = 15
Equation of curve is \( y = x + x^2 - x^4 + 15 \)

8 \int_{-1}^{1} (x^2 + 1) \, dx = \left[ \frac{x^3}{3} + x \right]_{-1}^{1}
= \left( \frac{3^3}{3} + 3 \right) - \left( -\frac{1}{3} - 1 \right)
= 12 + 1 = \frac{40}{3}

9 Area = \left[ \int_{1}^{x} (8x - x^2) \, dx = \left[ 4x^2 - \frac{x^3}{3} \right]_{1}^{3} \right]
= (4 \times 9 - 9) - \left( 4 - \frac{1}{3} \right) = 27 - 3\frac{2}{3} = 23\frac{1}{3}

10 You need to find the points of intersection first.
\quad x^2 + x - 1 = 9x - x^2 - 7
\Rightarrow 2x^2 - 8x + 6 = 0
\Rightarrow x^2 - 4x + 3 = 0
\Rightarrow (x - 1)(x - 3) = 0
\Rightarrow x = 1 \text{ giving, for both curves, } y = 1\
\text{and } x = 3 \text{ giving, for both curves, } y = 11
It is always useful to sketch the graphs of the curves.
Area = \int \left( (9x - x^3 - 7) - (x^2 + x - 1) \right) dx

\[= \int \left( -2x^2 + 8x - 6 \right) dx \]

\[= \left[ -\frac{2}{3}x^3 + 4x^2 - 6x \right]_3 \]

\[= \left( -\frac{2}{3} \cdot 3^3 + 4 \cdot 3^2 - 6 \cdot 3 \right) - \left( -\frac{2}{3} + 4 - 6 \right) \]

\[= (-18 + 36 - 18) - (-\frac{2}{3} + 2) \]

\[= 2\frac{2}{3} \]

11 (i) Use \( v^2 = u^2 + 2as \) with \( v = 30, \ u = 15 \) and \( s = 250 \).

\[ \Rightarrow a = \frac{30^2 - 15^2}{2 \times 250} = 1.35 \]

So the acceleration is \( 1.35 \text{ m/s}^2 \)

(iii) It is possible to use any of the \( svut \) formulae which includes the time for this part.

For example, using \( v = u + at \)

\[ 30 = 15 + 1.35t \]

\[ \Rightarrow t = \frac{15}{1.35} = 11.1 \text{ secs} \]

12 (i) The straight line indicates constant acceleration.

(ii) Acceleration = rate of change of velocity.

On a velocity-time graph the acceleration is the gradient of the line.

In this case

\[ \text{Acceleration} = \frac{15}{10} = 1.5 \text{ m/s}^2 \]

(iii) The distance travelled is the area under the graph.

In this case the distance travelled is the area under the line which is a triangle

\[ \text{Distance travelled} = \frac{1}{2} \times 15 \times 10 = 75 \text{ m} \]

13 (i) Use \( v^2 = u^2 + 2as \) where \( u = 18, \ a = -g, \ v = 0 \)

\[ s = \frac{18^2}{2g} = 16.5 \]

(ii) Use \( v = u + at \) with \( u = 18, \ v = 0, \ a = -g \)

\[ \Rightarrow 18 = gt \]

\[ \rightarrow t = \frac{18}{9.8} = 1.837 \text{ secs.} \]

The ball takes as long to fall as it did to reach its highest point, so total time = \( 1.837 \times 2 = 3.7 \), correct to 1 d.p.

14 (i) \[ v = 4t - t^2 \]

\[ \Rightarrow a = \frac{dv}{dt} = 4 - 2t \]

When \( t = 5, \ a = 4 - 10 = -6 \)

The particle is slowing down.
(ii) \( v = 4t - t^2 \)

\[ s = \int (4t - t^2) \, dt = 2t^2 - \frac{t^3}{3} + c \]

Given that \( s = 0 \) when \( t = 0 \) \( \Rightarrow c = 0 \)

When \( t = 5 \), \( s = 50 - \frac{125}{3} = \frac{81}{3} \)

So the displacement at 5 seconds is \( \frac{81}{3} \) m

(iii) \( s = 2t^2 - \frac{t^3}{3} \)

When \( s = 0 \), \( 2t^2 - \frac{t^3}{3} = 0 \)

\[ \Rightarrow t^2 = 0 \quad \text{and} \quad 2 - \frac{t^3}{3} = 0 \Rightarrow t = 6 \]

So the time when the particle is back at 0 is 6 seconds.

15 (i) \( v = 0.09r^2 - 0.002r^3 \)

\[ a = \frac{dv}{dr} = 0.18r - 0.006r^2 \]

\( a = 0 \) when \( t = 0 \) and also \( a = 0 \) when \( 0.18 - 0.006t = 0 \)

\[ \Rightarrow t = \frac{0.18}{0.006} = 30 \]

\[ \Rightarrow v = 0.09 \times 30^2 - 0.002 \times 30^3 \]

\[ v = 81 - 54 = 27 \]

So \( t = 30 \) seconds and then \( v = 27 \text{ m s}^{-1} \)

(ii) \( v = 0.09r^2 - 0.002r^3 \)

\[ s = \int ((0.09r^2 - 0.002r^3) \, dr = 0.03r^3 - 0.0005r^4 + c \]

(When \( t = 0 \), \( s = 0 \) \( \Rightarrow c = 0 \))

When \( t = 30 \), \( s = 0.03 \times 30^3 - 0.0005 \times 30^4 = 405 \text{ m} \)

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**Chapter 14 Differentiation**

**14.1 Differentiation and gradients of curves**

**Exam-style question (page 97)**

(i) \( y = x^3 + 3x^2 - 3x + 7 \)

\[ \frac{dy}{dx} = 3x^2 + 6x - 3 \]

When \( x = 2 \), \( \frac{dy}{dx} = 3 \times 2^2 + 6 \times 2 - 3 = 21 \)

(ii) \[ \frac{dy}{dx} = 3x^2 + 6x - 3 = -7 \]

\[ 3x^2 + 6x + 4 = 0 \]

\[ "b^2 - 4ac" = 6^2 - 4 \times 3 \times 4 < 0 \]

So no real roots showing that at no point is \( \frac{dy}{dx} = -7 \)
14.2 Tangents and normals

Exam-style question (page 98)

(i) The line has equation $2y - x + 9 = 0$

\[ y = \frac{x - 9}{2} \]

The curve meets the line when $x^2 - 4x - 1 = \frac{x - 9}{2}$

\[ 2x^2 - 8x - 2 = x - 9 \]

\[ 2x^2 - 9x + 7 = 0 \]

\[ 2x^2 - 2x - 7x + 7 = 0 \]

\[ 2x(x - 1) - 7(x - 1) = 0 \]

\[ (2x - 7)(x - 1) = 0 \]

Either $x = 1$ giving $y = -4$

or $x = \frac{7}{2}$ giving $y = -\frac{11}{4}$

\[ \text{coordinates are } (1, -4) \text{ and } \left(\frac{7}{2}, -\frac{11}{4}\right) \]

(ii) The line has gradient $\frac{1}{2}$

For the curve $y = x^2 - 4x - 1$

\[ \frac{dy}{dx} = 2x - 4 \]

When $x = 1$, \[ \frac{dy}{dx} = 2 - 4 = -2 \]

When $x = \frac{7}{2}$, \[ \frac{dy}{dx} = 2 \cdot \frac{7}{2} - 4 = 3 \]

To be a normal at either point requires the tangent to have gradient $-2$. So the line is a normal to the curve at the point $(1, -4)$ but not at $\left(\frac{7}{2}, -\frac{11}{4}\right)$.

14.3 Stationary points and the second derivative

Exam-style question (page 101)

(i) $y = x^3 - 3x^2 - 9x + 7$

\[ \frac{dy}{dx} = 3x^2 - 6x - 9 \]

\[ = 0 \text{ when } 3x^2 - 6x - 9 = 0 \]

\[ x^2 - 2x - 3 = 0 \]
\[(x - 3)(x + 1) = 0 \Rightarrow x = -1 \text{ or } 3\]

When \(x = -1\), \(y = -1 - 3 + 9 + 7 = 12\)
So the coordinates of the other turning point is \((-1, 12)\)

(iii) Method 1: use values of the function
\(f(-1.1) = 11.9, \ f(-1) = 12, \ f(-0.9) = 11.9\)
This indicates a maximum point as the values either side of the point as numerically smaller.
Method 2: use values of the gradient
\[\frac{dy}{dx} = 3x^2 - 6x - 9\]
When \(x = -1.1\), \(\frac{dy}{dx} = 1.23\)
When \(x = -0.9\), \(\frac{dy}{dx} = -1.17\)
The gradient is positive, goes to 0 and then negative indicates a maximum.
Method 3: use the second derivative
\[\frac{d^2y}{dx^2} = 6x - 6\]
At \(x = -1\), \(\frac{d^2y}{dx^2} = -6 - 6 = -12\)
The second derivative < 0 so this turning point is a maximum.

To find the coordinates of the other turning point, substitute \(x = 3\) into the equation \(y = 3x^3 - 3x^2 - 9x + 7 = -20\)
Then find the intercept on the y-axis which is when \(x = 0\) giving \(y = 7\)

Chapter 15 Integration

15.1 Integration

Exam-style question (page 103)

(i) \(\int (2x+2) \, dx = x^2 + 2x + c\)

(ii) \(\frac{dy}{dx} = 2x + 2\)
\[\Rightarrow y = x^2 + 2x + c\]
Through (3, 0)
\[
0 = 9 + 6 + c \\
\Rightarrow c = -15 \\
\Rightarrow y = x^2 + 2x - 15
\]

15.2 Definite integrals and area
Exam-style question (page 107)

(i) 
\[
x^2 - 4x + 6 = 4x - x^2 \\
\Rightarrow 2x^2 - 8x + 6 = 0 \\
\Rightarrow x^2 - 4x + 3 = 0 \\
\Rightarrow (x-1)(x-3) = 0 \\
\Rightarrow x = 1 \text{ or } 3
\]

(ii)  
Method 1
\[
A = \int_1^3 (4x - x^2) \, dx - \int_1^3 (x^2 - 4x + 6) \, dx \\
= \left[ \frac{2x^2 - x^3}{3} \right]_1^3 - \left[ \frac{x^3}{3} - 2x^2 + 6x \right]_1^3 \\
= \left( \frac{18 - 9}{3} \right) - \left( \frac{2 - \frac{1}{3}}{3} \right) - \left( \frac{9 - 18 + 18}{3} - \frac{1}{3} - 2 + 6 \right) \\
= \left( 9 - \frac{5}{3} \right) - \left( 9 - \frac{13}{3} \right) = \frac{22}{3} - \frac{14}{3} = \frac{8}{3}
\]

The area is \(2 \frac{2}{3}\) square units.

Method 2
\[
A = \int_1^3 (4x - x^2) \, dx - \int_1^3 (x^2 - 4x + 6) \, dx \\
= \int_1^3 \left( (4x - x^2) - (x^2 - 4x + 6) \right) \, dx \\
= \int_1^3 (8x - 2x^2 - 6) \, dx
\]
\[
\begin{align*}
4x^2 - 2x^3 - 6x &= (36 - 18 - 18) - \left(4 \cdot \frac{2}{3} - 6\right) \\
&= 0 - \left(\frac{8}{3}\right) = \frac{8}{3}
\end{align*}
\]

Then integrate the single resulting function.

The area is \(2\frac{2}{3}\) square units.

### Chapter 16 Applications to kinematics

#### 16.1 Constant acceleration

**Exam-style question (page 109)**

(i) Using \(v = u + at\), with \(v = 20\), \(u = 0\) and \(t = 8\) gives

\[
20 = 0 + 8a \\
\Rightarrow a = \frac{20}{8} = 2.5
\]

The acceleration is 2.5 m\(s^{-2}\).

(ii) Using \(s = \frac{u + v}{2}t\)

with \(u = 0\) and \(v = 20\) gives

\[
s = \left(\frac{0 + 20}{2}\right) \times 8 = 80
\]

The distance travelled is 80 m.

(iii) Using \(s = ut + \frac{1}{2}at^2\)

with \(s = 45\), \(u = 0\) and \(a = 2.5\) gives

\[
45 = 0 + \frac{1}{2} \times 2.5 \times t^2
\]

\[
t^2 = \frac{2 \times 45}{2.5}
\]

\[
t = \pm \sqrt{36} = 6 \text{ or } -6
\]

So the car takes 6 seconds to travel 45 metres.

(The negative time is not relevant to the situation.)

#### 16.2 Variable acceleration

**Exam-style question (page 111)**

(i) \(a = 2 + t\)

\[
\Rightarrow v = \int a\,dt = 2t + t^2 + c.
\]

When \(t = 0\), \(v = 4\) \(\Rightarrow c = 4\)

\[
\Rightarrow v = 2t + t^2 + 4.
\]

Integrate to find a function for \(v\).

Given the initial velocity, find the value of \(c\).
When \( t = 5 \), \( v = 10 + \frac{\frac{25}{2} + 4}{2} = 26 \frac{1}{2} \) \( \Rightarrow \) Substitute \( t = 5 \)

\[(iii) \quad s = \int v \, dt = t^2 + \frac{t^3}{6} + 4t + c \]

When \( t = 0 \), \( s = 0 \Rightarrow c = 0 \)

\[\Rightarrow s = t^2 + \frac{t^3}{6} + 4t \]

When \( t = 5 \), \( s = 25 + \frac{125}{6} + 20 = 65 \frac{5}{6} \) metres. \( \Rightarrow \) Integrate again to find a function for \( s \).

\[s = 0\] when \( t = 0 \) will find \( c \).

Review questions (Chapters 14–16) (pages 112–113)

1. \( y = x^2 - 2x + 7 \)
   \[\Rightarrow \frac{dy}{dx} = 2x - 2 \]
   When \( \frac{dy}{dx} = 4 \), \( 2x - 2 = 4 \)
   \[\Rightarrow x = 3 \]
   \[\Rightarrow y = 10 \]
   So coordinates are \((3, 10)\) \( \Rightarrow \) Differentiate.

2. \( \frac{dy}{dx} = 2x^3 - x^2 + 5 \)
   When \( x = 1 \), \( \frac{dy}{dx} = 6 \)
   So gradient of tangent = 6 \( \Rightarrow \) Set = 4 and solve.

   \[\Rightarrow \text{gradient of normal} = -\frac{1}{6} \]
   \[\Rightarrow y - 2 = -\frac{1}{6}(x - 1) \]
   \[\Rightarrow 6y - 12 = 1 - x \]
   \[\Rightarrow 6y + x = 13 \]
   Find the gradient of the tangent.

   \[\Rightarrow \text{Don’t forget to find the } y\text{-coordinate.} \]

   \[\Rightarrow \text{Use } m_1m_2 = -1. \]

   You can use the formula for the line as \( y = mx + c \) but in this case, given a point and the gradient the form \( y - y_1 = m(x - x_1) \) is better.

   \[\Rightarrow \text{Don’t leave the equation like this – simplify to three terms.} \]

3. (i) \( y = x^3 + 2x^2 - 5x + 5 \)
   \[\Rightarrow \frac{dy}{dx} = 3x^2 + 4x - 5 \]
   When \( x = 1 \), \( \frac{dy}{dx} = 3 + 4 - 5 = 2 \)
   \[\Rightarrow y - 3 = 2(x - 1) \]
   \[\Rightarrow y = 2x + 1 \]
   \[\Rightarrow \text{Solve simultaneously by substitution.} \]

   \[\Rightarrow \text{You know that the line is a tangent to the curve when}\]
   \[\Rightarrow x = 1 \text{ so two roots of this cubic equation are } 1 \text{ and } 1. \]

   \[\Rightarrow \text{So the line meets the curve again at } x = -4. \]
   \[\Rightarrow y = -7 \]

   Coordinates of \( R: (-4, -7) \)

(ii) \( y = x^3 + 2x^2 - 5x + 5 \) meets \( y = 2x + 1 \)
when \( x^3 + 2x^2 - 5x + 5 = 2x + 1 \)
\[\Rightarrow \quad x^3 + 2x^2 - 7x + 4 = 0 \]
\[\Rightarrow (x - 1)(x - 1)(x + 4) = 0 \]
So the line meets the curve again at \( x = -4 \).
\[\Rightarrow y = -7 \]

Coordinates of \( R: (-4, -7) \)
(iii) The gradient of the line is \( m = 2 \)
At \( R \), \[
\frac{dy}{dx} = 3x - 12x + 9
\]
So gradient of normal \( \neq \frac{1}{27} \)
So the answer is no, this line is not a normal to the curve at \( R \).

4 (i) \[
y = x^3 - 6x^2 + 9x + 5
\]
\[
\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9
\]
\[
\frac{dy}{dx} = 0 \quad \text{when} \quad 3x^2 - 12x + 9 = 0
\]
\[
\Rightarrow x^2 - 4x + 3 = 0
\]
\[
\Rightarrow (x - 3)(x - 1) = 0
\]
So stationary points when \( x = 1, 3 \)
When \( x = 1 \), \( y = 9 \)
When \( x = 0.9 \), \( \frac{dy}{dx} = 3(x - 3)(x - 1) = -ve \times -ve = +ve \)
When \( x = 1.1 \), \( \frac{dy}{dx} = -ve \times +ve = -ve \)
So at \((1, 9)\) the stationary value is a maximum.

(ii) The other stationary value occurs when \( x = 3 \).
Then \( y = 5 \)
that is, \((3, 5)\).

(iii)

There are three methods to determine the nature of this turning point. This method finds whether the gradient is +ve or −ve either side of the stationary point.
The other two methods are (i) to find the values of \( y \) either side, (ii) find the second derivative.

5 (i) \[
y = x^3 + \frac{3}{2}x^2 - 6x + 4
\]
\[
\Rightarrow \frac{dy}{dx} = 3x^2 + 3x - 6
\]
\[
\frac{dy}{dx} = 0 \quad \text{when} \quad 3x^2 + 3x - 6 = 0
\]
\[
\Rightarrow x^2 + x - 2 = 0
\]
\[
\Rightarrow (x + 2)(x - 1) = 0
\]
\[
\Rightarrow x = -2 \quad \text{giving} \quad y = 14
\]
and \( x = 1 \) giving \( y = \frac{1}{2} \)
(ii) \[
\frac{dy}{dx} = 3x^2 + 3x - 6
\]
\[
\Rightarrow \frac{d^2y}{dx^2} = 6x + 3
\]
when \(x = -2\), \(\frac{d^2y}{dx^2} < 0\) giving maximum
when \(x = 1\), \(\frac{d^2y}{dx^2} > 0\) giving minimum

(iii) \[
\frac{dy}{dx} = 2 + 2x - x^2
\]
\[
\Rightarrow y = 2x + x^2 - \frac{x^3}{3} + c
\]
Passes through \((3, 10)\) \(\Rightarrow 10 = 6 + 9 + c\)
\(\Rightarrow c = 4\)
\(\Rightarrow y = 2x + x^2 - \frac{x^3}{3} + 4\)

6. \[
\int (4 - x^2) \, dx
\]
\[
\int \left[4x - \frac{x^3}{3}\right]_2^4 = \left[\frac{16}{3} - \frac{16}{3}\right] = \frac{32}{3}
\]
This shows the method of using the second derivative. \(\frac{d^2y}{dx^2} < 0\) means that \(\frac{dy}{dx}\) is a decreasing function. So it was positive, is zero and will be negative giving a maximum.

Show the stationary points found above and also the intercept on the \(y\)-axis \((0, 4)\).

7. The curve meets the \(x\)-axis when
\(4 - x^2 = 0\)
\(\Rightarrow x = 2, -2\)
Area = \[
\int_{-2}^{2} (4 - x^2) \, dx
\]
\[
= \left[4x - \frac{x^3}{3}\right]_2^4
\]
\[
= \left[8 - \frac{8}{3}\right] - \left[8 - \frac{8}{3}\right]
\]
\[
= \frac{16}{3} - \frac{16}{3} = \frac{32}{3}
\]
Integrate.
Include the constant of integration.
Substitute the point given to find \(c\).
Find the limits.
Integrate.
8 (i) The curves meet when
\[ x^2 - 7x + 8 = -x^2 + 9x - 6 \]
\[ \Rightarrow 2x^2 - 16x + 14 = 0 \]
\[ \Rightarrow x^2 - 8x + 7 = 0 \]
\[ \Rightarrow (x - 7)(x - 1) = 0 \]
\[ \Rightarrow x = 1 \text{ giving } y = 2 \]
and \( x = 7 \) giving \( y = 8 \)
So A is at (1, 2) and B is at (7, 8)

(ii) Area \( = \int_{1}^{7} \left( (-x^2 + 9x - 6) - (-x^2 + 7x + 8) \right) \, dx \)
\[ = \int_{1}^{7} (-2x^2 + 16x - 14) \, dx \]
\[ = \left[ -\frac{2x^3}{3} + 8x^2 - 14x \right]_{1}^{7} \]
\[ = \left( -\frac{686}{3} + 392 - 98 \right) - \left( -\frac{2}{3} + 8 - 14 \right) \]
\[ = \frac{196}{3} \left( -\frac{20}{3} \right) = 216 \times \frac{3}{7} = 72 \]

9 (i) \( u = 20, a = 1.2 \)
Use \( v = u + at \)
\[ \Rightarrow v = 20 + 1.2 \times 5 = 26 \]
Speed = 26 m s\(^{-1}\)

(ii) Use \( s = ut + \frac{1}{2}at^2 \)
\[ \Rightarrow s = 20 \times 5 + \frac{1}{2} \times 1.2 \times 5^2 = 115 \]
Distance travelled = 115 m

10 In \( t \) s, A travels 13\( t \)
So B travels 13\( t \) + 16 in \( t \) s
B has acceleration \( a = \frac{1}{2} \) and \( u = 13 \)
The distance travelled by B is also given by:
\[ s = 13t + \frac{1}{2} \times \frac{1}{2} \times t^2 \]
\[ \Rightarrow 13t + 16 = 13t + \frac{t^2}{4} \]
\[ \Rightarrow \frac{t^2}{4} = 64 \]
\[ \Rightarrow t = 8 \]
Car B is 8 m in front of car A after 8 s.

11 (i) \( v = 0.36t^2 - 0.024t^3 \)
\[ \Rightarrow a = \frac{dv}{dt} = 0.72t - 0.072t^2 \]
\( v = 0.36t^2 - 0.024t^3 \)

\[ s = \int (0.36t^2 - 0.024t^3) dt \]

\[ = 0.12t^3 - 0.006t^4 + c \]

Given that the speed boat starts from rest, \( s = 0 \) when \( t = 0 \)

\[ \Rightarrow c = 0 \]

\[ \Rightarrow s = 0.12t^3 - 0.006t^4 \]

When \( t = 10 \), \( s = 120 - 60 = 60 \)

The boat travels 60 m.