

Chapter

8

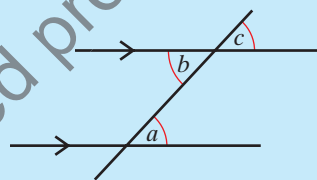
Congruency

This chapter is about

- Recognising and working with congruent triangles

You should already know

- The angle facts for parallel lines: alternate angles, a and b , are equal and corresponding angles, a and c are equal



- The angles in a triangle add up to 180°
- The sum of the angles on a straight line is 180°
- The exterior angle of a triangle is equal to the sum of the opposite, interior angles

Congruent triangles

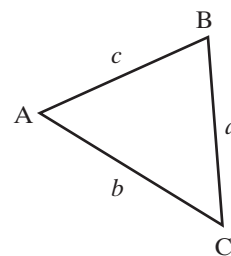
In Chapter 4 you learned that two shapes are congruent if they are exactly the same shape and size.

This means that they will fit exactly on to each other when one of them is rotated, reflected or translated.

Discovery 8.1

For each of the following triangles.

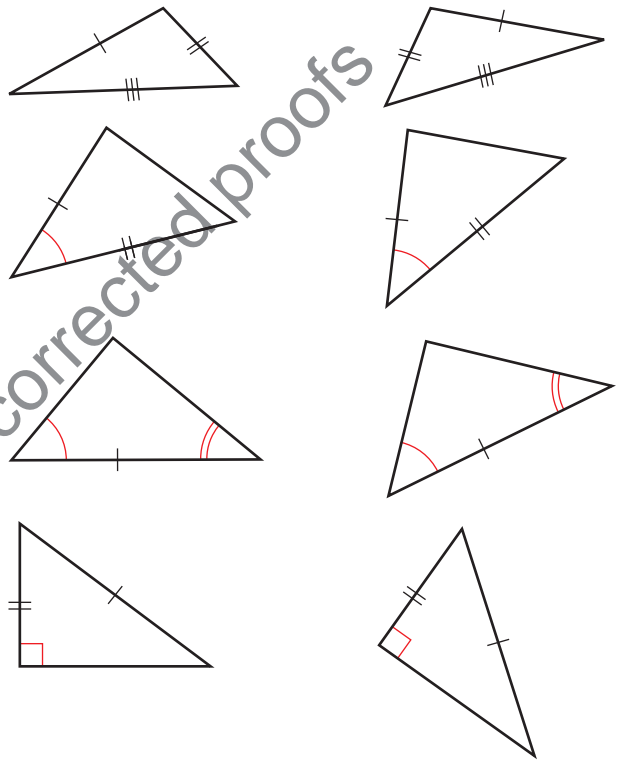
- Construct the triangle.
You will need to use compasses for some of them.
- Compare each of your triangles with your neighbours'.
Are they all congruent?



- (a) $a = 4 \text{ cm}, b = 5 \text{ cm}, c = 6 \text{ cm}$
- (b) $A = 50^\circ, b = 5 \text{ cm}, c = 6 \text{ cm}$
- (c) $A = 50^\circ, B = 60^\circ, c = 6 \text{ cm}$
- (d) $A = 90^\circ, a = 10 \text{ cm}, b = 6 \text{ cm}$
- (e) $A = 20^\circ, a = 6 \text{ cm}, b = 10 \text{ cm}$

Two triangles are congruent if any of these conditions are satisfied.

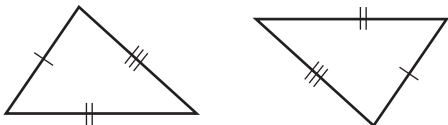
- The three sides of one triangle are equal to the corresponding three sides of the other triangle (side, side, side or SSS).
- Two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle (side, angle, side or SAS).
- Two angles and the side between them of one triangle are equal to two angles and the side between them in the other triangle (angle, side, angle or ASA).
- Each triangle is right-angled and the hypotenuse and one other side of one triangle is equal to the hypotenuse and one side of the other triangle (right angle, hypotenuse, side or RHS).



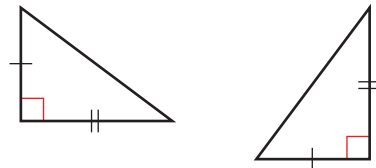
Example 8.1

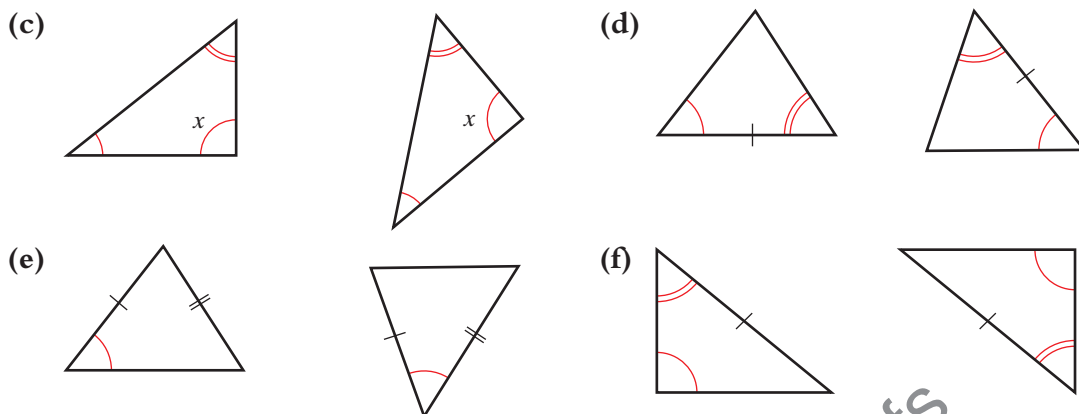
State whether or not each of these pairs of triangles are congruent. If the triangles are congruent, give a reason for your answer.

(a)



(b)





Solution

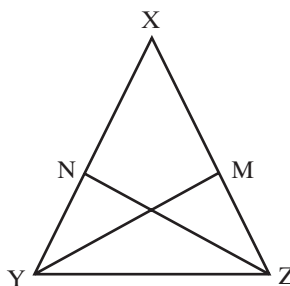
- (a) Congruent: SSS
- (b) Congruent: SAS
- (c) Not congruent
- (d) Congruent: ASA
- (e) Not congruent
- (f) Congruent: the missing third angles are also equal, ASA

Proof 8.1

Triangle XYZ is isosceles with $XY = XZ$.

The bisector of angle Y meets XZ at M; the bisector of angle Z meets XY at N.

Prove that $YM = ZN$.



Solution

Angle XYZ = angle XZY

so
angle MYZ = angle NZY.

In triangles YMZ and ZNY, YZ is common.

So triangles YMZ and ZNY are congruent. ASA

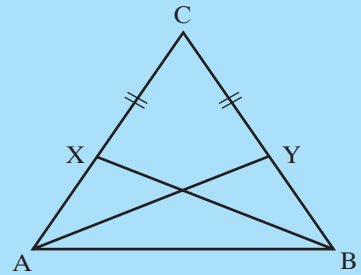
So $YM = ZN$.

Base angles of an isosceles triangle are equal.

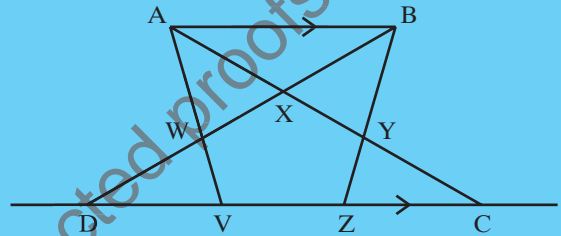
Since YM bisects angle XYZ and ZN bisects angle XZY.

Exercise 8.1

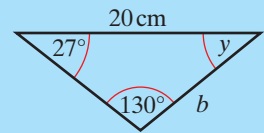
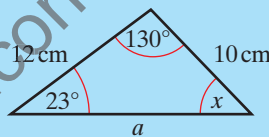
- 1 ABC is an isosceles triangle with $AC = BC$.
 X and Y are points on AC and BC so that $CX = CY$.
 Prove that triangle CXB is congruent to triangle CYA .



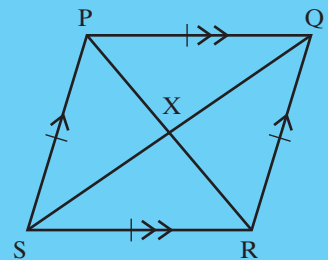
- 2 In this diagram, $DX = XC$, $DV = ZC$ and AB is parallel to DC .
 Prove that triangle DBZ is congruent to triangle CAV .



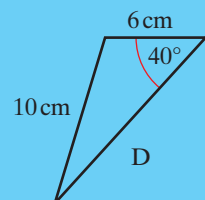
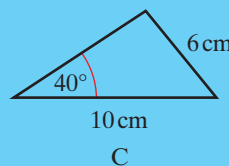
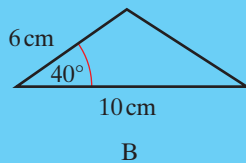
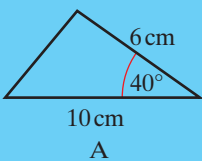
- 3 These two triangles are congruent.
 (a) Write down the length of sides a and b .
 (b) Write down the size of angles x and y .



- 4 $PQRS$ is a rhombus. $PQ = QR = RS = SP$.
 PQ is parallel to RS , QR is parallel to PS .
 (a) Prove that
 (i) triangles PQX and RSX are congruent.
 (ii) triangles PSX and QRX are congruent.
 (b) Prove that X is the midpoint of SQ and of PR , and hence that the diagonals of a rhombus intersect at right angles.



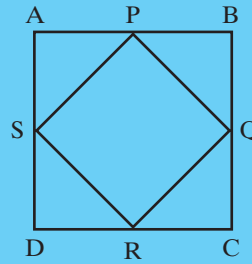
- 5 Which two of these triangles are congruent?



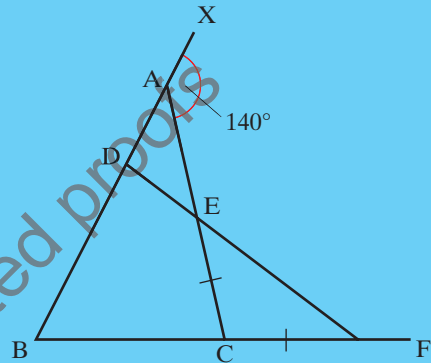
Exercise continues ...



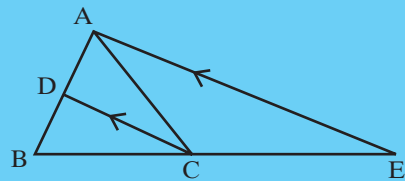
- 6 ABCD is a square. P, Q, R and S are the midpoints of the sides.
Prove that the quadrilateral PQRS is a square.



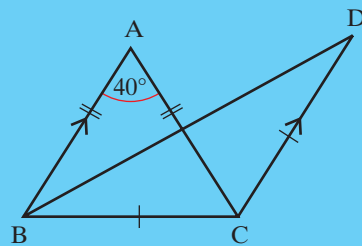
- 7 In the diagram, $AB = AC$, angle $XAE = 140^\circ$ and $CE = CF$.
Find angle ADE , giving reasons for your answer.



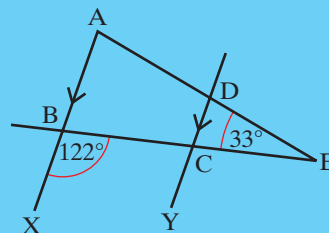
- 8 In this diagram, CD is the bisector of angle ACB and AE is parallel to DC.
Prove that $AC = CE$.



- 9 In this diagram, $AB = AC$, CD is parallel to BA, $CD = CB$ and angle $BAC = 40^\circ$.
Calculate the size of angle DBC , giving reasons for your answer.



- 10 In this diagram, angle $CBX = 122^\circ$, angle $DEC = 33^\circ$ and AX is parallel to DY.
Calculate the size of angles BAD and BCY , giving reasons for your answers.

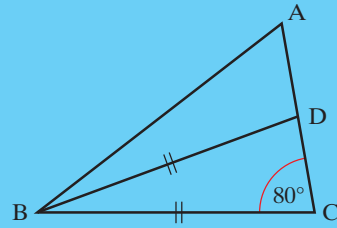


Exercise continues ...



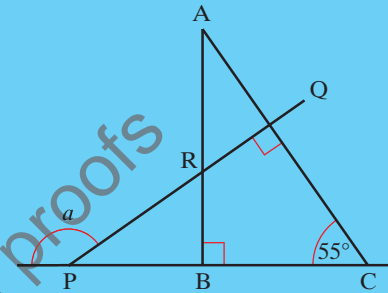
- 11 In triangle ABC, BD bisects angle ABC, angle ACB = 80° and $BD = BC$.

Prove that angle BAC = 60° .



- 12 ABC is a right-angled triangle. Angle ACB = 55° and angle PQC = 90° .

Prove that angle $a = 145^\circ$.



Challenge 8.1

- Draw a triangle with one side 6 cm and angles 35° , 66° and 79° . Compare your triangle with your neighbours. Are they all congruent? If the answer is 'No', how many different triangles are there? Are all the triangles similar?

Challenge 8.2

- Plane figures other than triangles, for example, rectangles and octagons, can be congruent. Two rectangles have the same area and the same perimeter. Does this mean that they must be congruent? Explain your answer.

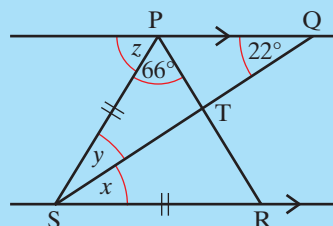
What you have learned

- Two shapes are congruent if they are exactly the same shape and size; corresponding sides are equal in length and corresponding angles are equal in size
- Two triangles are congruent if one of these four statements is true:
 - the corresponding three sides of each triangle are equal (SSS)
 - two sides and the included angle in each triangle are equal (SAS)
 - two angles and the side between them in each triangle are equal (ASA)
 - each triangle is right-angled and the hypotenuse and one other side in each triangle is equal (RHS)

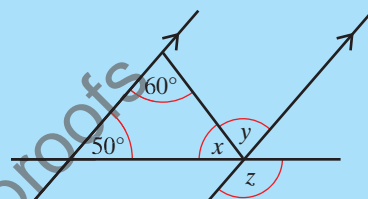
Mixed exercise 8

- 1 In this diagram PQ is parallel to SR, $SP = SR$, angle $SPR = 66^\circ$ and angle $PQS = 22^\circ$.

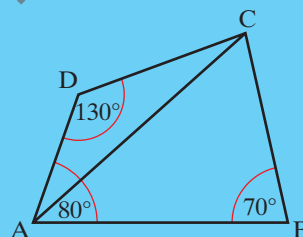
Find the size of angles x , y , z , giving reasons for your answers.



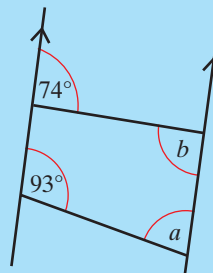
- 2 Calculate the size of angles x , y and z , giving reasons for your answers.



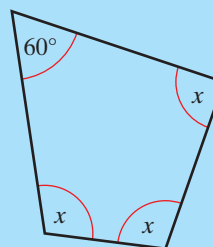
- 3 ABCD is a quadrilateral. Angle $CBA = 70^\circ$, angle $BAD = 80^\circ$ and angle $ADC = 130^\circ$. CA bisects angle DCB. Prove that triangle CAB is isosceles.



- 4 Find the size of angles a and b , giving reasons for your answers.



- 5 Find the size of x , giving reasons for your answer.



- 6 Use congruent triangles to prove that the diagonals of a kite cross at right angles.
- 7 Two straight lines, AB and CD, bisect each other at right angles at E. Prove that the straight lines AC, CB, BD and DA are equal in length.