TOPIC 1

Scalars and vectors

Introduction

The topic of scalars and vectors has applications in all parts of a physics specification. As such, the topic is one that should be taught at an early stage in the course. It should not be treated as a 'sideline' that appears in different areas.

The experience of examiners is that where a question is set specifically on this topic, students do not perform well. They can, however, take components in particular directions, for example application of the motor effect formula $F = BIL \sin\theta$. This does cast doubt on whether many do understand what they are doing. So-called applications may be a matter of rote-learning.

The range of examples of vectors used in the development of the topic is limited. This is intentional, since it is expected that this topic will be taught early in the course when students' knowledge of vector quantities is restricted.

Background knowledge

Students are not expected to have any background knowledge beyond their GCSE studies.

When students are introduced to scalars and vectors, they should have knowledge of some quantities that may be used as examples. As the AS/A-level course progresses, the topic will be reinforced by further examples.

Learning outcomes

After they have finished this topic, students should:

1 understand what is meant by 'scalar' and 'vector' quantities
2 be able to give examples of scalar and vector quantities
3 be able to use a vector triangle to determine the resultant of two coplanar vectors
4 be able to calculate the resultant of two perpendicular vectors
5 understand how to resolve a vector into two perpendicular components
6 understand that the perpendicular components of a vector are independent of one another

Lesson notes

question

An ant walks in a straight line so that it moves through a distance of 1 cm in every 3 s. It starts at point P. Where will it be after 12 s?

answer

The answer may be either '4 cm from P' or 'don't know'.

The conclusion is that all we can tell is that the ant will be 4 cm from P, but anywhere on a circle of radius 4 cm, with its centre at P.

To find the position, we need the *direction* of movement.

Development

Scalars and vectors

Some quantities can be fully defined by giving only the magnitude — **scalar quantities**.

Other quantities must be specified with a magnitude and a direction — **vector quantities**.

Ask students for examples of scalars and vectors, and draw up a list.

	Scalar	Vector
Mass	•	
Force		•
Weight		•
Volume	•	
Density	•	
Speed	•	
Velocity		•
Acceleration		•
Electrical resistance	•	
Energy	•	

Weight is a vector, despite the fact that we do not usually state its direction. Weight is a force and we assume it acts towards the centre of the Earth.

Representation of a vector

Draw the vector as an arrow. The length of the arrow represents its magnitude (size) and the arrow points in the direction of the vector.

A scale for the magnitude must be indicated. Students should use a ruler and, when necessary, a set square and protractor. Many marks in examinations are lost through careless drawings.

example

A bird flies horizontally due west at a speed of $4.0\,m\,s^{-1}$. The wind is blowing due south at $3.0\,m\,s^{-1}$.

Draw vectors to represent these two velocities.

answer

See Figure 1.1.

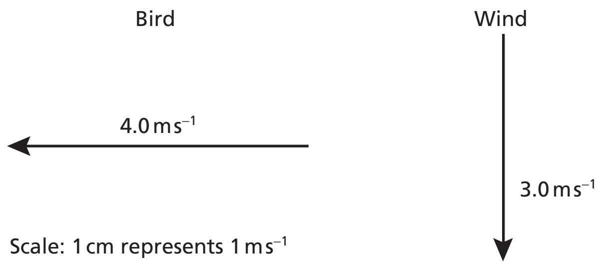


Figure 1.1

Resultant of two vectors

Some people on the ground watch the bird. In which direction and at what speed do they see the bird flying?

The combined effect of two or more vectors is known as the **resultant**.

The resultant is found using a **vector triangle**.

If the two vectors are represented in magnitude and direction by the sides of a triangle, then the third side represents, in magnitude and direction, the resultant.

A scale diagram is required.

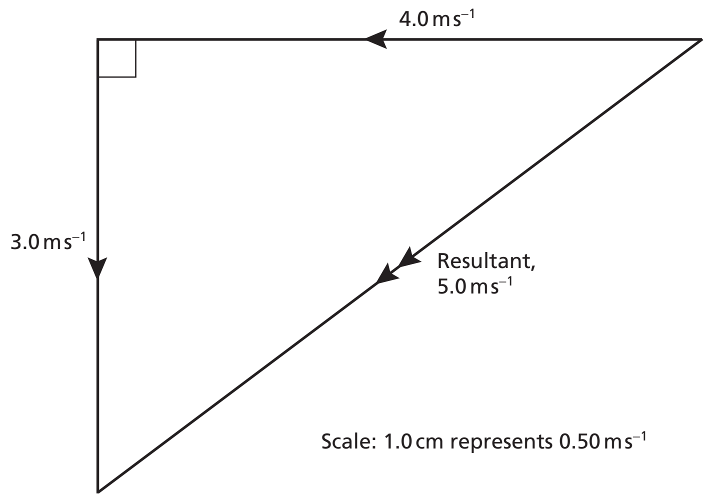


Figure 1.2
A vector triangle

The two component vectors must be either clockwise or anticlockwise. That is, they must be in the same direction.

The direction of the resultant is found by joining the 'starting point' of the first vector to the 'finishing point' of the second vector.

Experiment

To justify the use of a vector triangle

- Attach three light strings to a small metal ring.
- Tie the free ends of the strings to newtonmeters.
- Rub the strings with coloured chalk and place the strings over a sheet of white paper.
- Pull on the strings so that ring does not move.

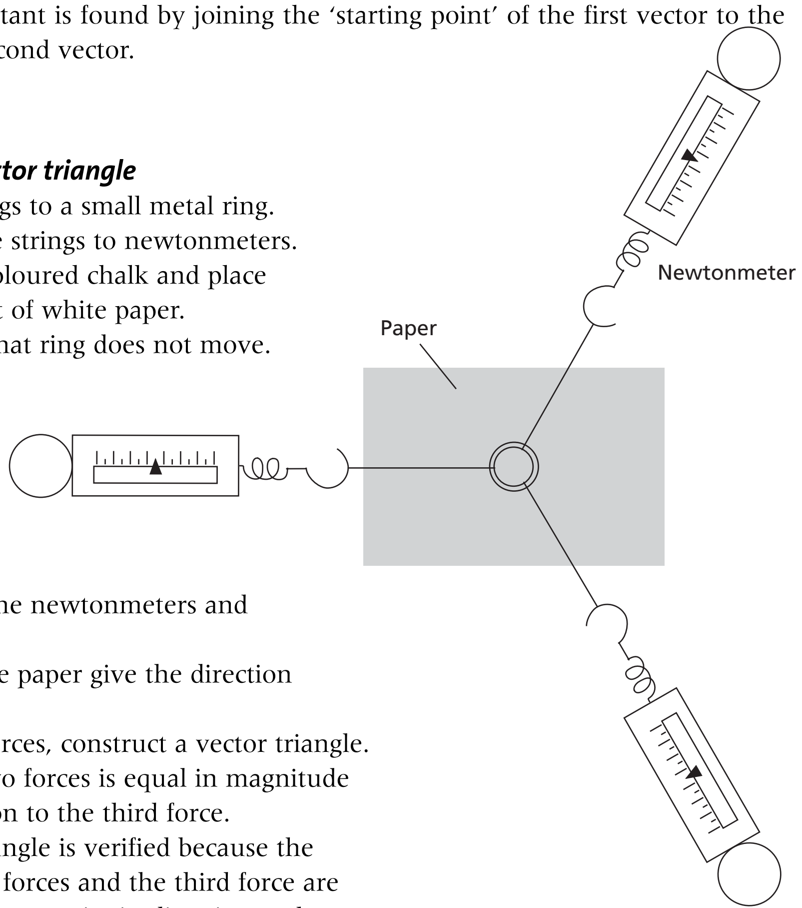


Figure 1.3
Apparatus to justify the triangle of vectors

- Note the readings on the newtonmeters and flick the strings.
- The chalk marks on the paper give the direction of each force.
- Choosing two of the forces, construct a vector triangle.
- The resultant of the two forces is equal in magnitude but opposite in direction to the third force.
- The use of a vector triangle is verified because the resultant of two of the forces and the third force are equal in magnitude but opposite in direction and so the ring would not move — it is in **equilibrium**.

Students must give the argument as to *why* the use of the triangle is justified.

The experiment can be carried out using weights and pulleys, as shown in Figure 1.4.

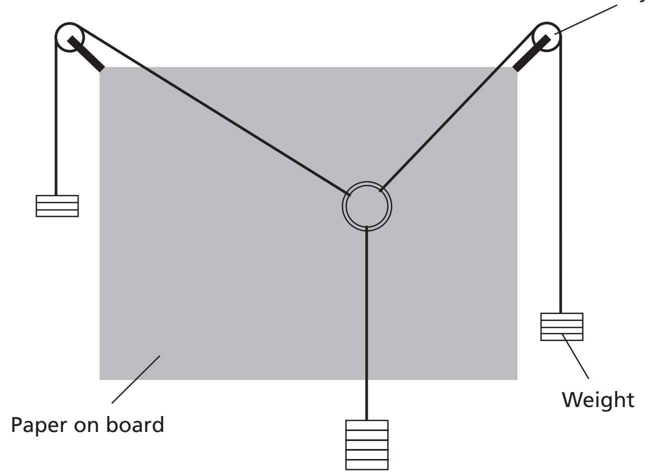


***Figure 1.4** Alternative arrangement of apparatus*

example

A cricket ball is thrown by a player. At one instant, the ball is moving horizontally to the right at 8.0 m s^{-1} and vertically upwards at 5.5 m s^{-1}.

Draw a vector diagram to determine the resultant velocity of the ball.

answer

See Figure 1.5.

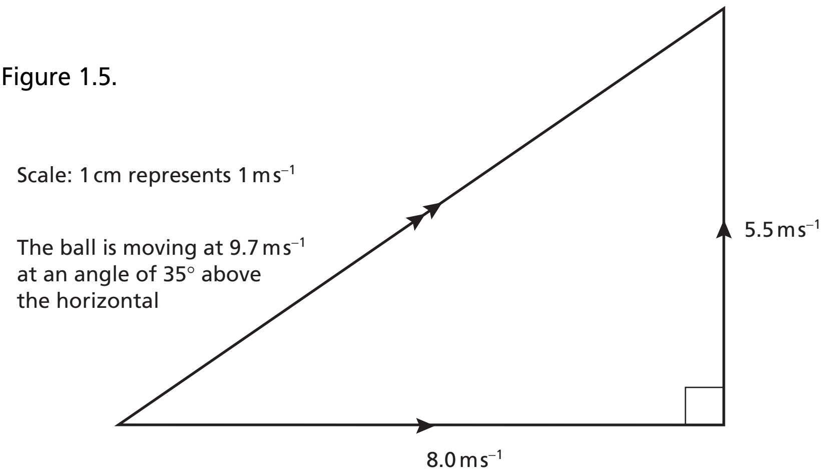


Figure 1.5

Scale drawing takes time and has limited precision and accuracy. So why not *sketch* the vector triangle and then *calculate* the resultant velocity?

In this example, by Pythagoras's theorem

$$R^2 = 8.0^2 + 5.5^2 = 94.3$$
$$R = 9.7\,\text{m s}^{-1}$$

and

$$\tan\theta = 5.5/8.0 = 0.6875$$
$$\theta = 34.5° \approx 35°$$

The ball is moving at 9.7 m s^{-1} to the right at an angle of 35° above the horizontal.

Students frequently fail to give a statement of the answer.

Resolution of a vector into two perpendicular components

If two vectors can have a single resultant, then a single vector can be **resolved** into two perpendicular **components**.

This can be achieved either by drawing a scale diagram or by calculation.

It is essential that students are able to draw scale diagrams, but given the choice, it is usually quicker and more accurate to sketch a vector diagram and then complete the problem by calculation.

example

A ball is thrown upwards and, at one instant, has a velocity of $16\,\mathrm{m\,s^{-1}}$ at an angle of 25° above the horizontal. Determine the horizontal and vertical components of the velocity.

answer

By drawing: see Figure 1.6

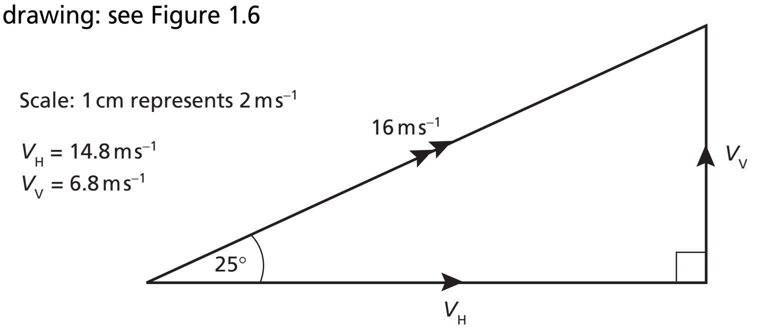


Figure 1.6

By calculation:

Use the scale drawing as the sketch.

$V_H = 16 \cos 25° = 14.5\,\mathrm{m\,s^{-1}} \approx 15\,\mathrm{m\,s^{-1}}$

$V_V = 16 \sin 25° = 6.76\,\mathrm{m\,s^{-1}} \approx 6.8\,\mathrm{m\,s^{-1}}$

Referring to Figure 1.7, in general, we can write

$V_H = V \cos\theta$

$V_V = V \sin\theta$

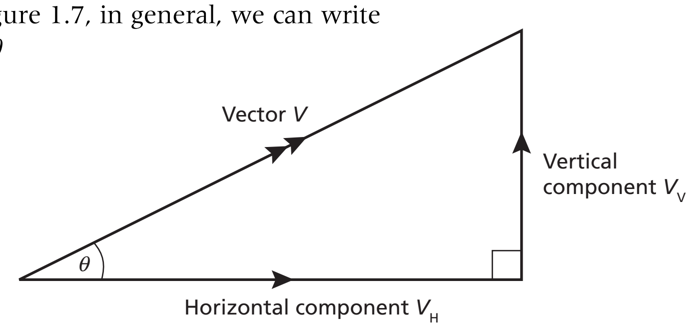


Figure 1.7 Resolving a vector into two perpendicular components

Why bother to resolve vectors?

- The perpendicular components of a vector are independent of one another.
- Calculations can be performed more easily because one direction can be considered at a time.

Answers to worksheet

1 1.3 m s^{-1} travelling downstream at an angle of 67° to the bank

2 51N between the two forces and at an angle of 61° to the 25 N force

3 **(a)** (i) 9.0 m s^{-1}
(ii) 53° below the horizontal

4 **(a)** (i) 53 N
(ii) 37 N

5 **(a)** (i) 15 N
(ii) 32 N

Scalars and vectors

A **scalar quantity** has magnitude only.

A **vector quantity** has magnitude and direction.

Examples:

	Scalar	Vector
Mass	•	
Force		•
Weight		•
Volume	•	
Density	•	
Speed	•	
Velocity		•
Acceleration		•
Electrical resistance	•	
Energy	•	

A vector is represented by an arrow pointing in the direction of the vector. The length of the arrow represents the magnitude (size).

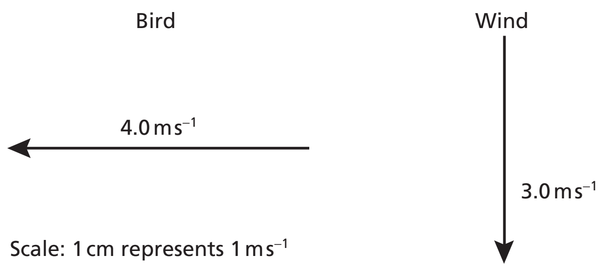
A scale drawing is needed.

example

A bird flies horizontally due west at a speed of 4.0 m s^{-1}. The wind is blowing due south at 3.0 m s^{-1}.

Draw vectors to represent these two velocities.

answer



The combined effect of two or more vectors is known as the **resultant**.

The resultant is found using a **vector triangle**.

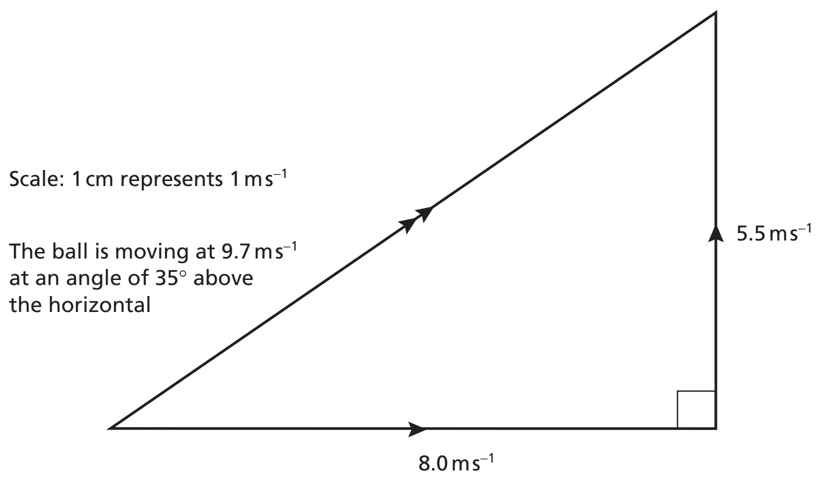
If the two vectors are represented in magnitude and direction by the sides of a triangle, then the third side represents, in magnitude and direction, the resultant.

example

A cricket ball is thrown by a player. At one instant, the ball is moving horizontally to the right at $8.0\,m\,s^{-1}$ and vertically upwards at $5.5\,m\,s^{-1}$.

Draw a vector diagram to determine the resultant velocity of the ball.

answer



The answer may also be found by calculation.

In this example, by Pythagoras's theorem

$$R^2 = 8.0^2 + 5.5^2 = 94.3$$
$$R = 9.7\,m\,s^{-1}$$

and

$$\tan\theta = 5.5/8.0 = 0.6875$$
$$\theta = 34.5° \approx 35°$$

The ball is moving at $9.7\,m\,s^{-1}$ to the right at an angle of 35° above the horizontal.

Always *sketch* a vector triangle and give a statement of the answer.

A single vector can be **resolved** into two perpendicular **components**.

example

A ball is thrown upwards and, at one instant, has a velocity of 16 m s^{-1} at an angle of 25° above the horizontal.

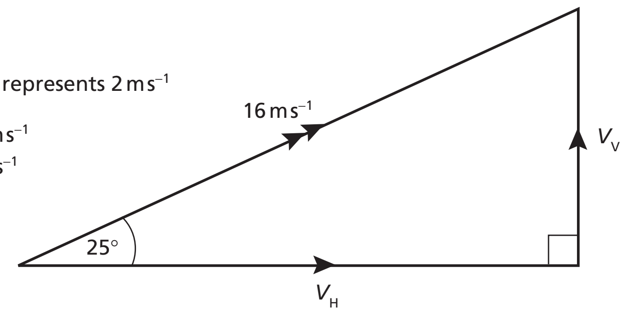
Determine the horizontal and vertical components of the velocity.

answer

By drawing:

Scale: 1 cm represents 2 m s^{-1}

V_H = 14.8 m s^{-1}
V_V = 6.8 m s^{-1}



By calculation:

Use the scale drawing as the sketch.

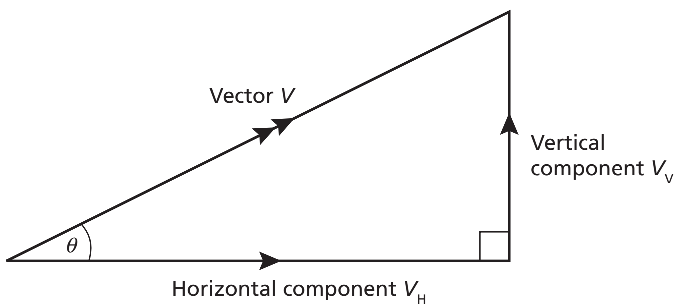
$V_H = 16 \cos 25° = 14.5 \text{ m s}^{-1} \approx 15 \text{ m s}^{-1}$

$V_V = 16 \sin 25° = 6.76 \text{ m s}^{-1} \approx 6.8 \text{ m s}^{-1}$

Referring to the diagram below, in general, we can write

$V_H = V \cos\theta$

$V_V = V \sin\theta$



Why bother to resolve vectors?

- The perpendicular components of a vector are independent of one another.
- Calculations can be performed more easily because one direction can be considered at a time.

Questions

1 A river is flowing at a constant speed of 0.50 m s^{-1}. A person in a rowing boat rows at a speed of 1.2 m s^{-1} normal to the bank of the river.

(a) Use a scale drawing to determine the resultant velocity of the boat.

(b) Check your answer to **(a)** by calculation.

2 Forces of 25 N and 45 N act on an object. The angle between the forces is 90°.

(a) Use a scale drawing to determine the resultant force on the object.

(b) Check your answer to **(a)** by calculation.

3 A person watches a stone as it falls from a cliff. At one instant in the motion, he notes that the stone is travelling at a speed of 15 m s^{-1}. He calculates that the vertical speed of the stone is 12 m s^{-1}.

(a) Use a scale drawing to determine:
(i) the horizontal speed of the ball
(ii) the angle at which the ball is travelling to the horizontal

(b) Check your answer to **(a)** by calculation.

4 A person pushes a wheelbarrow with a force of 65 N. The force acts downwards at an angle of 55° to the vertical.

(a) Use a scale drawing to determine:
(i) the horizontal component of the force
(ii) the vertical component of the force

(b) Check your answers to **(a)** by calculation.

5 A block of weight 35 N rests on a rough slope. The slope makes an angle of 25° to the horizontal.

(a) Use a scale drawing to determine the component of the weight:
(i) down the slope
(ii) normal to the slope

(b) Check your answers to **(a)** by calculation.