

# Chapter 5

## Ratio and proportion

### This chapter is about

- Understanding ratio and its notation
- Writing a ratio in its lowest terms
- Writing a ratio in the form  $1:n$
- Using ratios in proportion calculations
- Dividing a quantity in a given ratio
- Comparing proportions

### You should already know

- How to find common factors
- How to simplify fractions
- What is meant by an enlargement
- How to change between metric units

## What is a ratio?

A ratio is used to compare two or more quantities.

If you have three sweets and decide to keep one and give two to your best friend, you and your friend have sweets in the ratio  $1:2$ . You say this as '1 to 2'.

Larger numbers can also be compared in a ratio.

If you have six sweets and decide to keep two and give four to your best friend, you and your friend have sweets in the ratio  $2:4$ .

You already know how to give a fraction in its lowest terms, by cancelling. You can do the same with ratios.

$2:4 = 1:2$     2 and 4 are both multiples of 2. So you can divide each part of the ratio by 2.

### Example 5.1

The salaries of three people are £16 000, £20 000 and £32 000. Write this as a ratio in its lowest terms.

### Solution

$$\begin{aligned} & 16\,000 : 20\,000 : 32\,000 && \text{First write the salaries as a ratio.} \\ = & 16 : 20 : 32 && \text{Divide each part of the ratio by 1000.} \\ = & 8 : 10 : 16 && \text{Divide each part by 2.} \\ = & 4 : 5 : 8 && \text{Divide each part by 2.} \end{aligned}$$

Notice that your answer should not include units. £4 : £5 : £8 would be wrong.

### TIP

To write a ratio in its lowest terms in one step, find the highest common factor (HCF) of the numbers in the ratio. Then divide each part of the ratio by the HCF.





## Solution

$$\begin{aligned} \text{(a)} \quad 50\text{p} : \pounds 2 &= 50\text{p} : 200\text{p} \\ &= 1 : 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 2\text{ cm} : 6\text{ mm} &= 20\text{ mm} : 6\text{ mm} \\ &= 10 : 3 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 600\text{ g} : 2\text{ kg} : 750\text{ g} &= 600\text{ g} : 2000\text{ g} : 750\text{ g} \\ &= 12 : 40 : 15 \end{aligned}$$

Write each part in the same units.  
Divide each part by 50.

Write each part in the same units.  
Divide each part by 2.

Write each part in the same units.  
Divide each part by 50.

## Exercise 5.1

1 Write each of these ratios in its lowest terms.

(a) 6 : 3

(b) 25 : 75

(c) 30 : 6

(d) 5 : 15 : 25

(e) 6 : 12 : 8

2 Write each of these ratios in its lowest terms.

(a) 50 g : 1000 g

(b) 30p : £2

(c) 2 minutes : 30 seconds

(d) 4 m : 75 cm

(e) 300 ml : 2 litres

3 At a concert there are 350 men and 420 women.

Write the ratio of men to women in its lowest terms.

4 Al, Peta and Dave invest £500, £800 and £1000 respectively in a business.

Write the ratio of their investments in its lowest terms.

5 A recipe for vegetable soup uses 1 kg of potatoes, 500 g of leeks and 750 g of celery.

Write the ratio of the ingredients in its lowest terms.

(a) Explain why the ratio 20 minutes : 1 hour is not 20 : 1.

(b) What should it be?

## Challenge 5.1

(a) Explain why the ratio 20 minutes : 1 hour is not 20 : 1.

(b) What should it be?

## Writing a ratio in the form 1 : n

It is sometimes useful to have a ratio with 1 on the left.

A common scale for a scale model is 1 : 24.

The scale of a map or enlargement is often given as 1 : n.

To change a ratio to this form, divide both numbers by the one on the left. This can be written in a general form as 1 : n.





### Example 5.6

- (a) A negative is enlarged in the ratio 1 : 20 to make a picture.  
The negative measures 36 mm by 24 mm.  
What size is the enlargement?
- (b) Another 1 : 20 enlargement measures 1000 mm  $\times$  1000 mm.  
What size is the negative?

### Solution

- (a)  $36 \times 20 = 720$  The enlargement will be 20 times bigger than  
 $24 \times 20 = 480$  the negative, so multiply both dimensions by 20.  
The enlargement measures 720 mm by 480 mm.
- (b)  $1000 \div 20 = 50$  The negative will be 20 times smaller than the  
negative, so divide the dimensions by 20.  
The negative measures 50 mm  $\times$  50 mm.

### Example 5.7

A map is drawn to a scale of 1 cm : 2 km.

- (a) On the map, the distance between Anhope and Didburn is 5.4 cm.  
What is the actual distance in kilometres?
- (b) The length of a straight railway track between two stations is 7.8 km.  
How long is this track on the map in centimetres?

### Solution

- (a)  $2 \times 5.4 = 10.8$  The actual distance, in kilometres, is  
Real distance = 10.8 km twice as large as the map distance, in  
centimetres. So multiply by 2.
- (b)  $7.8 \div 2 = 3.9$  The map distance, in centimetres, is  
Map distance = 3.9 cm half as large as the actual distance, in  
kilometres. So divide by 2.

### Challenge 5.2

What would the answer to part (a) of Example 5.7 be in centimetres?  
What ratio could you use to work this out?



Sometimes you have to work out quantities using a ratio that is not in the form  $1:n$ .

To work out an unknown quantity, you multiply each part of the ratio by the same number to get an equivalent ratio which contains the quantity you know. This number is called the **multiplier**.

### Example 5.8

To make jam, fruit and sugar are mixed in the ratio  $2:3$ .

This means that if you have 2 kg of fruit, you need 3 kg of sugar; if you have 4 kg of fruit, you need 6 kg of sugar.

How much sugar do you need if your fruit weighs

- (a) 6 kg?                      (b) 10 kg?                      (c) 500 g?

### Solution

(a)  $6 \div 2 = 3$

$$2:3 = 6:9$$

9 kg of sugar

(b)  $10 \div 2 = 5$

$$2:3 = 10:15$$

15 kg of sugar

(c)  $500 \div 2 = 250$

$$2:3 = 500:750$$

750 g of sugar

Divide the quantity of fruit by the fruit part of the ratio to find the multiplier.

Multiply each part of the ratio by the multiplier, 3.

Divide the quantity of fruit by the fruit part of the ratio to find the multiplier.

Multiply each part of the ratio by the multiplier, 5.

Divide the quantity of fruit by the fruit part of the ratio to find the multiplier.

Multiply each part of the ratio by the multiplier, 250.

### Example 5.9

Two photos are in the ratio  $2:5$ .

- (a) What is the height of the larger photo?  
(b) What is the width of the smaller photo?

5 cm



9 cm



## Solution

- (a)  $5 \div 2 = 2.5$  Divide the height of the smaller photo by the smaller part of the ratio to find the multiplier.  
 $2:5 = 5:12.5$  Multiply each part of the ratio by the multiplier, 2.5.  
 Height of the larger photo = 12.5 cm
- (b)  $9 \div 5 = 1.8$  Divide the width of the larger photo by the larger part of the ratio to find the multiplier.  
 $2:5 = 3.6:9$  Multiply each part of the ratio by the multiplier, 1.8.  
 Width of the smaller photo = 3.6 cm

## Example 5.10

To make grey paint, white paint and black paint are mixed in the ratio 5:2.

- (a) How much black paint is mixed with 800 ml of white paint?  
 (b) How much white paint is mixed with 300 ml of black paint?

## Solution

A table is often useful for this sort of question.

Paint	White	Black
Ratio	5	2
(a) Amount	800 ml	$2 \times 160 = 320$ ml
Multiplier	$800 \div 5 = 160$	
(b) Amount	$5 \times 150 = 750$ ml	300 ml
Multiplier		$300 \div 2 = 150$

## TIP

Make sure you haven't made a silly mistake by checking that the bigger side of the ratio has the bigger quantity.

(a) Black paint = 320 ml

(b) White paint = 750 ml



### Example 5.11

To make stew for four people, a recipe uses 1.6 kg of beef.  
How much beef is needed using the recipe for six people?

### Solution

The ratio of people is 4:6.

$$4:6 = 2:3$$

Write the ratio in its lowest terms.

$$1.6 \div 2 = 0.8$$

Divide the quantity of beef needed for four people by the first part of the ratio to find the multiplier.

$$0.8 \times 3 = 2.4$$

Multiply the second part of the ratio by the multiplier, 0.8.

Beef needed for six people = 2.4 kg

### Exercise 5.3

- The ratio of the lengths of two squares is 1:6.
  - The length of the side of the small square is 2 cm. What is the length of the side of the large square?
  - The length of the side of the large square is 21 cm. What is the length of the side of the small square?
- The ratio of helpers to babies in a crèche must be 1:4.
  - There are six helpers on a Tuesday. How many babies can there be?
  - There are 36 babies on a Thursday. How many helpers must there be?
- Sanjay is mixing pink paint. To get the shade he wants, he mixes red and white paint in the ratio 1:3.
  - How much white paint should he mix with 2 litres of red paint?
  - How much red paint should he mix with 12 litres of white paint?
- The negative of a photo is 35 mm long. An enlargement of 1:4 is made. What is the length of the enlargement?
- A road atlas of Great Britain is to a scale of 1 inch to 4 miles.
  - On the map the distance between Forfar and Montrose is 7 inches. What is the actual distance between the two towns in miles?
  - It is 40 miles from Newcastle to Middlesbrough. How far is this on the map?

*Exercise continues ...*



- 6 For a recipe, Chelsy mixes water and lemon curd in the ratio 2 : 3.
- (a) How much lemon curd should she mix with 20 ml of water?
  - (b) How much water should she mix with 15 teaspoons of lemon curd?
- 7 To make a solution of a chemical a scientist mixes 3 parts chemical with 20 parts water.
- (a) How much water should he mix with 15 ml of chemical?
  - (b) How much chemical should he mix with 240 ml of water?
- 8 An alloy is made by mixing 2 parts silver with 5 parts nickel.
- (a) How much nickel must be mixed with 60 g of silver?
  - (b) How much silver must be mixed with 120 g of nickel?
- 9 Sachin and Rehan share a flat. They agree to share the rent in the same ratio as their wages.  
Sachin earns £600 a month and Rehan earns £800 a month.  
If Sachin pays £90, how much does Rehan pay?
- 10 A recipe for hotpot uses onions, carrots and stewing steak in the ratio, by mass, of 1 : 2 : 5.
- (a) What quantity of steak is needed if 100 g of onion is used?
  - (b) What quantity of carrots is needed if 450 g of steak is used?



### Discovery 5.1

Maya has an evening job making up party bags for a children's party organiser.

She shares out lemon sweets and raspberry sweets in the ratio 2 : 3.  
Each bag contains 5 sweets.

- (a) On Monday Maya makes up 10 party bags.
    - (i) How many sweets does she use in total?
    - (ii) How many lemon sweets does she use?
    - (iii) How many raspberry sweets does she use?
  - (b) On Tuesday Maya makes up 15 party bags.
    - (i) How many sweets does she use in total?
    - (ii) How many lemon sweets does she use?
    - (iii) How many raspberry sweets does she use?
- What do you notice?



A ratio represents the number of shares in which a quantity is divided. The total quantity divided in a ratio is found by adding the parts of the ratio together.

To find the quantities shared in a ratio:

- Find the total number of shares.
- Divide the total quantity by the total number of shares to find the multiplier.
- Multiply each part of the ratio by the multiplier

**TIP**

The multiplier may not be a whole number. Work with the decimal or fraction and round the final answer if necessary.

**Example 5.12**

To make fruit punch, orange juice and grapefruit juice are mixed in the ratio 5 : 3.

Jo wants to make 1 litre of punch.

- (a) How much orange juice does she need in millilitres?  
 (b) How much grapefruit juice does she need in millilitres?

**Solution**

$5 + 3 = 8$  First work out the total number of shares.  
 $1000 \div 8 = 125$  Convert 1 litre to millilitres and divide by 8 to find the multiplier.

A table is often helpful for this sort of question.

Punch	Orange	Grapefruit
Ratio	5	3
Amount	$5 \times 125 = 625$ ml	$2 \times 125 = 375$ ml

- (a) Orange juice = 625 ml      (b) Grapefruit juice = 375 ml

**TIP**

To check your answers, add the parts together: they should equal the total quantity. For example,  $625 \text{ ml} + 375 \text{ ml} = 1000 \text{ ml}$  ✓

**Exercise 5.4**

Do not use your calculator for questions 1 to 5.

- Share £20 between Dave and Sam in the ratio 2 : 3.
- Paint is mixed in the ratio 3 parts red to 5 parts white to make 40 litres of pink paint.
 

(a) How much red paint is used?      (b) How much white paint is used?
- Asif is making mortar by mixing sand and cement in the ratio 5 : 1.  
 How much sand is needed to make 36 kg of mortar?

*Exercise continues ...*



4 To make a solution of a chemical a scientist mixes 1 part chemical with 5 parts water. She makes 300 ml of the solution.

(a) How much chemical does she use? (b) How much water does she use?

5 Amit, Bree and Chris share £1600 between them in the ratio 2 : 5 : 3.  
How much does each receive?

You may use your calculator for questions 6 to 8.

6 In a local election, 5720 people vote.  
They vote for Labour, Conservative and other parties in the ratio 6 : 3 : 2.  
How many people vote Conservative?

7 St Anthony's College Summer Fayre raised £1750. The governors decided to share the money between the college and a local charity in the ratio 5 to 1.  
How much did the local charity receive? Give your answer correct to the nearest pound.

8 Sally makes breakfast cereal by mixing bran, currants and wheatgerm in the ratio 8 : 3 : 1 by mass.

(a) How much bran does she use to make 600 g of the cereal?

(b) One day, she only has 20 g of currants.  
How much cereal can she make? She has plenty of bran and wheatgerm.

### Challenge 5.3

Okera has a photo measuring 13 cm by 17 cm. He wants to have it enlarged.

Supa Print offer two sizes: 24 inches by 32 inches and 20 inches by 26.5 inches.

He wants to keep the same proportions, or as near as possible.

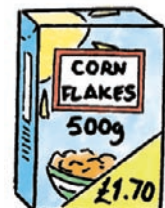
(a) Which of the two enlargements should he choose?  
Show how you make your decision.

(b) Why might he choose the other one?

### Discovery 5.2

Two packets of cornflakes are available at a supermarket.

Which is the better value for money?





To compare value, you need to compare either

- how much you get for a certain amount of money or
- how much a certain quantity (for example, volume or mass) costs.

In each case you are comparing **proportions**, either of size or of cost.

The better value item is the one with the **lower unit cost** or the **greater number of units per penny** (or pound).

### Example 5.13

Sunflower oil is sold in 700 ml bottles for 95p and in 2 litre bottles for £2.45.

Show which bottle is the better value.

### Solution

#### Method 1

Work out the price per millilitre for each bottle.

Size	Small	Large
Capacity	700 ml	2 litre = 2000 ml
Price	55p	£2.45 = 245p
Price per ml	$95 \div 700 = 0.14\text{p}$	$245 \div 2000 = 0.1225\text{p}$

Use the same units for each bottle.

Round your answers to 2 decimal places if necessary.

The price per ml of the 2 litre bottle is lower. It has the lower unit cost. In this case the unit is a millilitre.

The 2 litre bottle is the better value.

#### Method 2

Work out the amount per penny for each bottle.

Size	Small	Large
Capacity	700 ml	2 litre = 2000 ml
Price	95p	£2.45 = 245p
Amount per penny	$700 \div 95 = 7.37\text{p}$	$2000 \div 245 = 8.16\text{ml}$

Again use the same units for each bottle.

Round your answers to 2 decimal places if necessary.

The amount per pence is greater for the 2 litre bottle. It has the greater number of units per penny.

The 2 litre bottle is the better value.

### TIP

Make it clear whether you are working out the cost per unit or the amount per penny, and include the units in your answers. Always show your working.




## Exercise 5.5

- 1 A 420 g bag of Choco bars costs £1.59 and a 325 g bag of Choco bars costs £1.09. Which is the better value for money?
- 2 Spa water is sold in 2 litre bottles for 85p and in 5 litre bottles for £1.79. Show which is the better value.
- 3 Wallace bought two packs of cheese, a 680 g pack for £3.20 and a 1.4 kg pack for £5.40. Which was the better value?
- 4 One-inch nails are sold in packets of 50 for £1.25 and in packets of 144 for £3.80. Which packet is the better value?
- 5 Toilet rolls are sold in packs of 12 for £1.79 and in packs of 50 for £7.20. Show which is the better value.
- 6 Brillo white toothpaste is sold in 80 ml tubes for £2.79 and in 150 ml tubes for £5.00. Which tube is the better value?
- 7 A supermarket sells cola in three different sized bottles: a 3 litre bottle costs £1.99, a 2 litre bottle costs £1.35 and a 1 litre bottle costs 57p. Which bottle gives the best value?
- 8 Crispy cornflakes are sold in three sizes: 750 g for £1.79, 1.4 kg for £3.20 and 2 kg for £4.89. Which packet gives the best value?

## What you have learned

- To write a ratio in its lowest terms, divide all parts of the ratio by their highest common factor (HCF)
- To write a ratio in the form  $1:n$ , divide both numbers by the one on the left
- If the ratio is in the form  $1:n$ , you can work out the second quantity by multiplying the first by  $n$ , and you can work out the first quantity by dividing the second quantity by  $n$
- To find an unknown quantity, each part of the ratio must be multiplied by the same number, called the multiplier
- To find the quantities shared in a given ratio, first find the total number of shares, then divide the total quantity by the total number of shares to find the multiplier, then multiply each part of the ratio by the multiplier
- To compare value, work out the cost per unit or the number of units per penny (or pound). The better value item is the one with the lower cost per unit or the greater number of units per penny (or pound)

**Mixed exercise 5**

- 1** Write each ratio in its simplest form.
- (a) 50 : 35                      (b) 30 : 72                      (c) 1 minute : 20 seconds  
(d) 45 cm : 1 m                (e) 600 ml : 1 litre
- 2** Write these ratios in the form 1 : n.
- (a) 2 : 8                          (b) 5 : 12                          (c) 2 mm : 10 cm  
(d) 2 cm : 5 km                (e) 100 : 40
- 3** A notice is enlarged in the ratio 1 : 20.
- (a) The original is 3 cm wide.  
How wide is the enlargement?  
(b) The enlargement is 100 cm long.  
How long is the original?
- 4** To make 12 scones Maureen uses 150 g of flour.  
How much flour does she use to make 20 scones?
- 5** To make a fruit and nut mixture, raisins and nuts are mixed in the ratio 5 : 3, by mass.
- (a) What mass of nuts is mixed with 100 g of raisins?  
(b) What mass of raisins is mixed with 150 g of nuts?
- 6** Panache made a fruit punch by mixing orange, lemon and grapefruit juice in the ratio 5 : 1 : 2.
- (a) He made a 2 litre bowl of fruit punch.  
How many millilitres of grapefruit juice did he use?  
(b) How much fruit punch could he make with 150 ml of orange juice?
-  **7** Show which is the better buy: 5 litres of oil for £18.50 or 2 litres of oil for £7.00.
- 8** Supershop sells milk in pints at 43p and in litres at 75p.  
A pint is equal to 568 ml.  
Which is the better buy?