

Chapter 1

Integers

This chapter is about

- Adding, subtracting, multiplying and dividing integers
- Multiples and factors
- Rounding numbers to the nearest 10, 100, 1000, ...
- Squares, cubes and other powers
- Square roots
- Negative numbers

You should already know

- An integer is a whole number, for example 7, 18 or 253
- How to do simple additions, subtractions, multiplications and divisions
- Your multiplication tables up to the 10 times table

Arithmetic check

When you are doing calculations, write the numbers in columns: units under units, tens under tens, and so on.

Example 1.1


Work out these.

- (a) $46 + 32$ (b) $78 - 32$ (c) $38 + 126$ (d) $164 - 38$

Solution

- (a)
$$\begin{array}{r} 46 \\ + 32 \\ \hline 78 \end{array}$$
 Simply add the digits in each column.
 $6 + 2 = 8$ and $4 + 3 = 7$.
- (b)
$$\begin{array}{r} 78 \\ - 32 \\ \hline 46 \end{array}$$
 Simply subtract the digits in each column.
 $8 - 2 = 6$ and $7 - 3 = 4$.
- (c)
$$\begin{array}{r} 38 \\ + 126 \\ \hline 164 \\ 1 \end{array}$$
 $8 + 6 = 14$.
You write 4 in the units column and a small 1 at the bottom to show you are carrying 1 'ten' over from the units column to the tens column.
 $3 + 2 = 5$; $5 + 1$ carried over = 6.
- (d)
$$\begin{array}{r} 15\cancel{6}14 \\ - 38 \\ \hline 126 \end{array}$$
 You cannot take 8 from 4 so you change the 6 tens into 5 tens and 10 units.
 $14 - 8 = 6$ and $5 - 3 = 2$.

Challenge 1.2

-  (a) Look again at Example 1.2.
- (i) What other calculation can be made using the three numbers 3, 32 and 96?
- (ii) What other calculation can be made using the three numbers 7, 18 and 126?
- (b) $65 \times 6 = 390$. Write down two other calculations that can be made using these numbers.
- (c) Write down three calculations that can be made using the numbers 43, 20 and 860.

Exercise 1.1

1 Work out these.

(a) $46 + 53$

(b) $54 + 37$

(c) $78 + 46$

(d) $158 + 23$

(e) $136 + 282$

(f) $264 + 189$

2 Work out these.

(a) $96 - 55$

(b) $64 - 27$

(c) $75 - 28$

(d) $147 - 53$

(e) $236 - 129$

(f) $562 - 286$

3 Work out these.

(a) 23×3

(b) 19×4

(c) 36×5

(d) 68×7

(e) 123×6

(f) 262×4

4 Work out these.

(a) $84 \div 4$


(b) $72 \div 3$

(c) $75 \div 5$

(d) $91 \div 7$

(e) $144 \div 6$

(f) $184 \div 4$

-  5 Jamie bought a CD for £14, a pair of trainers for £38 and a ticket for a football match for £17. What was the total cost?
- 6 Jatindar was given £80 for her birthday. She bought some clothes for £53. How much did she have left?
- 7 Emma bought six packets of biscuits at 46p each. What was the total cost in pence? How much is the total in £s?
- 8 A school has £182 to spend on books. The books they want to buy cost £7 each. How many books can they buy?

Multiples

The numbers in the five times table are 5, 10, 15, 20, 25,

5, 10, 15, 20, 25, ... are called **multiples** of 5.

You should know your five times table up to '12 fives are 60' but the multiples of five do not stop at 60. They go on 65, 70, 75, In fact there is no end to the list of multiples.

Example 1.3

- (a) List the multiples of 2 that are less than 35.
- (b) List the multiples of 6 that are less than 100.
- (c) List the multiples of 9 that are less than 100.

Solution

You list the 2, the 6 and the 9 times tables and carry on until you get to 35 or 100, as instructed.

- (a) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34
- (b) 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
- (c) 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99

The multiples of two are also called the **even numbers**.

Notice that they all end in 0, 2, 4, 6 or 8.

So 1398 is an even number because it ends in 8.

All the other integers, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, ... are called **odd numbers**.

Notice that they all end in 1, 3, 5, 7 or 9.

So 6847 is an odd number because it ends in 7.

Look again at Example 1.3.

Notice that 18, 36, 54, 72 and 90 are in the list of multiples for both 6 and 9.

18, 36, 54, 72 and 90 are called **common multiples** of 6 and 9 because they are in, or common to, both lists.

Factors

Discovery 1.1

There are 70 sweets in a jar.

- (a) Can the sweets be shared equally between three people?
- (b) Find all the numbers of people the sweets can be shared between. How many do they each receive?

A number that will divide into a number exactly is called a **factor** of the number.

For example,

- 2 is a factor of 8,
- 7 is a factor of 21,
- 10 is a factor of 100,
- 1 is a factor of 6,
- 9 is a factor of 9.

Notice that 1 is a factor of every number and every number is a factor of itself.

Check up 1.1

Find all the factors of these numbers.

- (a) 12
- (b) 25
- (c) 48
- (d) 100

The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

The factors of 50 are 1, 2, 5, 10, 25 and 50.

Notice that 1, 2, 5 and 10 are in both lists of factors.

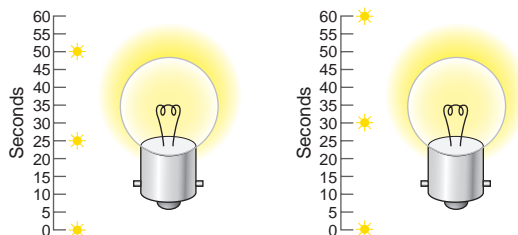
1, 2, 5 and 10 are called the **common factors** of 30 and 50 because they are common to both lists.

Challenge 1.3

One light flashes every 25 seconds.

Another light flashes every 30 seconds together.

How many seconds will it be before they flash together again?



TIP

Once you have gone beyond half the number, there will be no new factors except the number itself.

Rounding numbers

The Earth is approximately a sphere. It is not exact however and the radius varies from about 6356 km at the North Pole to 6378 km at the Equator.

The average radius is 6367 km.

Since it varies so much, an approximate value for the radius will do for most calculations.



6400 km is the usual approximation. It is accurate to the nearest 100 km.

For very large numbers it is usual to approximate to the nearest hundred, thousand, ten thousand, etc.

The distance from the Earth to the Sun varies but it is usually given as 93 million miles, that is 93 000 000, to the nearest million.

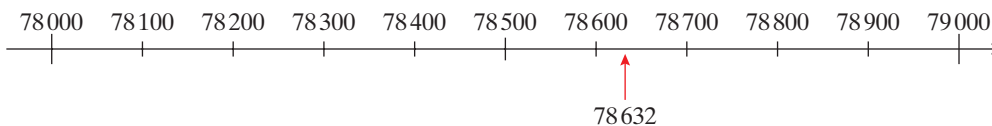
This headline appeared in a newspaper.

Local Man Wins £79 000!

This does not necessarily mean that the man won exactly £79 000.

The actual prize may have been £78 632 but the headline makes more impact if it is rounded to the nearest thousand.

Counting in thousands, 78 632 is between 78 000 and 79 000.



It is nearer to 79 000.

So 78 632 to the nearest thousand is 79 000.

Here is a quick method of rounding to the nearest thousand.

Step 1: Put a ring round the thousands digit. For example, 7⑧632.

Step 2: Look at the next digit to the right.

If it is less than 5, leave the thousands digit as it is.

If it is 5 or more, add 1 to the thousands digit.

Step 3: Replace the remaining digits by zeros. For the example above, 79 000.

A similar method can be used to round to the nearest 100, 10 000, etc.

Example 1.4

- (a) Round 45 240 to the nearest 100.
 (b) Round 458 000 to the nearest 10 000.
 (c) Round 6375 to the nearest 10.

Solution

- (a) $45 \textcircled{2}40$ 2 is the 100s digit.
 45 200 4 is less than 5.
- (b) $4\textcircled{5}8\ 000$ 5 is the 10 000s digit.
 460 000 8 is greater than 5.
- (c) $63\textcircled{7}5$ 7 is the tens digit.
 6380 5 is 5 or more.

Discovery 1.2

Find some examples of numbers in newspapers or magazines that have probably been rounded.

For each one, decide whether the number has been rounded to the nearest 100, 1000, million, etc.

Exercise 1.2

- List the following.
 - The multiples of 6 less than 100
 - The multiples of 8 less than 100
- Use your answers to question 1 to list the common multiples of 6 and 8 less than 100.
- Look at these numbers.
 2, 6, 15, 18, 30, 33
 - Which have 2 as a factor?
 - Which have 3 as a factor?
 - Which have 5 as a factor?
- List the following.
 - The multiples of 12 less than 100
 - The multiples of 15 less than 100
- Use your answers to question 4 to find a common multiple of 12 and 15 less than 100.

Exercise continues ...



Example 1.7

- (a) 300×40 (b) 42×30 (c) 54×40 (d) 27×500

Solution

- (a) First, you multiply 300×4 :

$$\begin{array}{r} 300 \\ \times 4 \\ \hline 1200 \end{array}$$

Then you need to multiply by 10.
You can do this by just adding 0 to your answer.

$$1200 \times 10 = 12\,000$$

- (b)
$$\begin{array}{r} 42 \\ \times 3 \\ \hline 126 \end{array}$$

$$126 \times 10 = 1260$$

- (c)
$$\begin{array}{r} 54 \\ \times 4 \\ \hline 216 \\ 1 \end{array}$$

$$216 \times 10 = 2160$$

- (d)
$$\begin{array}{r} 27 \\ \times 5 \\ \hline 135 \\ 1 \end{array}$$

$$135 \times 100 = 13\,500$$

TIP

A quick way to work out a calculation such as 300×40 is to do $3 \times 4 = 12$. Then count the number of zeros in the calculation, three in this case, and add them to your answer. So the answer is 12 000.

More difficult multiplications

You need to be able do questions like 53×38 or 258×63 without a calculator.

There are several methods to do this. Two are shown here but your teacher may show you more. Choose a method you are happy with and stick with it.

Method 1

$$\begin{array}{r} 53 \\ \times 38 \\ \hline 1590 \quad (53 \times 30) \\ 424 \quad (53 \times 8) \\ \hline 2014 \quad \text{Add} \\ 11 \end{array}$$

$$\begin{array}{r}
 258 \\
 \times 63 \\
 \hline
 153480 \quad (258 \times 60) \\
 71724 \quad (258 \times 3) \\
 \hline
 16254 \\
 \hline
 1 \quad 1
 \end{array}$$

TIP

63 × 258 would give the same answer as 258 × 63 but it is usually easier to have the smaller number on the bottom.

This is the traditional method, called 'long multiplication'. The second method uses a grid.

Method 2

53×38

×	50	3
30	1500	90
8	400	24

$$\begin{array}{r}
 1500 \\
 400 \\
 90 \\
 + 24 \\
 \hline
 2014 \\
 \hline
 1
 \end{array}$$

258×63

×	200	50	8
60	12000	3000	480
3	600	150	24

$$\begin{array}{r}
 12000 \\
 3000 \\
 480 \\
 600 \\
 150 \\
 + 24 \\
 \hline
 16254 \\
 \hline
 1 \quad 1
 \end{array}$$

Exercise 1.3

1 Work out these.

(a) 52×10

(b) 63×100

(c) 54×1000

(d) 361×100

(e) $56 \times 10\,000$

(f) 60×100

(g) 549×1000

(h) 8100×100

(i) 530×1000

(j) $47 \times 10\,000$

(k) $923 \times 100\,000$

(l) $62 \times 1\,000\,000$

2 Work out these.

(a) $530 \div 10$

(b) $14\,000 \div 100$

(c) $532\,000 \div 1000$

(d) $64\,000 \div 100$

(e) $6\,400\,000 \div 1000$

(f) $536\,000 \div 10$

(g) $675\,400 \div 100$

(h) $7\,300\,000 \div 100$

(i) $58\,000\,000 \div 10\,000$

Exercise continues ...

3 Work out these.

(a) 30×50

(b) 70×80

(c) 70×200

(d) 200×300

(e) 800×30

(f) 50×40

(g) 600×3000

(h) 600×500

(i) 800×7000

(j) 4000×3000

(k) $70\,000 \times 40$

(l) 9000×8000

4 Work out these.

(a) 64×30

(b) 72×60

(c) 234×30

(d) 56×200

(e) 63×400

(f) 78×300

(g) 432×600

(h) 58×4000

5 Work out these.

(a) 54×32

(b) 38×62

(c) 57×82

(d) 98×18

(e) 66×29

(f) 84×74

(g) 123×27

(h) 264×35

(i) 483×72

(j) 691×43

(k) 542×81

(l) 88×236

6 (a) How many pence are there in £632?

(b) Change 5600 pence into pounds.

7 1 kilometre is 1000 metres.

How many metres is 47 kilometres?

8 Trainers cost £40 per pair.

What do six pairs cost?

9 Gary walks 400 metres to school and 400 metres back.

How far does he walk in 195 school days?

- 10 28 people attended Rajvee's birthday party. She gave them each a packet of sweets which cost 34p each. What was the total cost in pence?



Squares and cubes

You read 5^2 as '5 squared'.

$$5^2 \text{ means } 5 \times 5 = 25.$$

All the **squares** from 1^2 to 10^2 are in your tables so you should know them.

TIP

It is a very common mistake to think that 5^2 means 5×2 .

For harder squares you can use your calculator.

Look for the square function on your calculator. The key may look like $\boxed{x^2}$ or $\boxed{\blacksquare}$

Example 1.8

Find 18^2 .

Solution

There are two ways of doing this on a calculator.

Method 1

Work out $18 \times 18 = 324$.

Method 2

Use the square function on your calculator.

Enter 18 and then press $\boxed{x^2}$ and $\boxed{=}$. The display should read 324.

You read 2^3 as '2 cubed'.

$$2^3 \text{ means } 2 \times 2 \times 2 = 8.$$

You should be able to work out 2^3 , 3^3 , 10^3 and possibly even 4^3 and 5^3 in your head but for other cubes you may need your calculator.

Some calculators do not have a 'cube' button, $\boxed{x^3}$, so it is probably best to use the $\boxed{\times}$ button twice.

Example 1.9

Work out 17^3 .

Solution

$$17 \times 17 \times 17 = 4913$$

Other powers

Squares and cubes are examples of **powers**. Another way of saying 2^2 is '2 to the power 2' and of saying 2^3 is '2 to the power 3'.

Squares and cubes are the only powers which have special names.

TIP

Calculators vary. Make sure you are familiar with the functions on your calculator.

TIP

It is a very common mistake to think that 2^3 means 2×3 .

You read 5^4 as '5 to the power 4'.

$$5^4 \text{ means } \underbrace{5 \times 5 \times 5 \times 5}_{\text{four fives multiplied together}} = 625$$

At this stage you will not need to find powers of most numbers other than squares or cubes.

The powers of ten, however, form a sequence which you already know.

Discovery 1.3

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$\text{Work out } 10^4 = 10 \times 10 \times 10 \times 10 =$$

Work out 105.

What do you notice about the power of 10 and the number of zeros?

Write down the value of (a) 106. (b) 108.

Square roots

Ashraf thought of a number and then multiplied it by itself.

The answer was 36.

What number did Ashraf start with? $? \times ? = 36$

Using your tables you should realise that Ashraf started with 6 because $6^2 = 36$.

Discovery 1.4

Work with a friend.

Take turns to think of a number and multiply it by itself. Tell your friend your answer. The other person must find what number you started with.

Continue until you cannot find any more.

What you have been doing in Discovery 1.4 is finding **square roots**. That is the reverse of finding squares.

$$\text{'Square root' is written } \sqrt{\quad} \text{ so } \sqrt{36} = 6.$$

For harder square roots you will need your calculator. Look for the $\sqrt{\square}$ button on your calculator.

Example 1.10

Find $\sqrt{289}$.

Solution

Press $\sqrt{\square}$ and then $\boxed{2}$ $\boxed{8}$ $\boxed{9}$ and then $\boxed{=}$.

The display should read 17.

Check that $17 \times 17 = 289$.

Exercise 1.4

1 Work out these without your calculator.

- | | | | | |
|---------|---------|----------|----------|----------|
| (a) 72 | (b) 92 | (c) 112 | (d) 122 | (e) 302 |
| (f) 502 | (g) 602 | (h) 2002 | (i) 4002 | (j) 8002 |

2 Use your calculator to work out these.

- | | | | | |
|---------|---------|----------|----------|----------|
| (a) 142 | (b) 222 | (c) 312 | (d) 472 | (e) 892 |
| (f) 562 | (g) 342 | (h) 1802 | (i) 2632 | (j) 7452 |

3 Use your calculator to work out these.

- | | | | |
|---------|---------|---------|----------|
| (a) 63 | (b) 93 | (c) 113 | (d) 143 |
| (e) 253 | (f) 373 | (g) 433 | (h) 1473 |

4 Use your calculator to work out these.

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| (a) $\sqrt{225}$ | (b) $\sqrt{196}$ | (c) $\sqrt{361}$ | (d) $\sqrt{529}$ |
| (e) $\sqrt{1521}$ | (f) $\sqrt{7569}$ | (g) $\sqrt{4624}$ | (h) $\sqrt{2916}$ |

5 Work out these without your calculator.

- | | | | | |
|------------------|------------------|-------------------|-------------------|----------------------|
| (a) $\sqrt{400}$ | (b) $\sqrt{900}$ | (c) $\sqrt{2500}$ | (d) $\sqrt{6400}$ | (e) $\sqrt{40\,000}$ |
|------------------|------------------|-------------------|-------------------|----------------------|

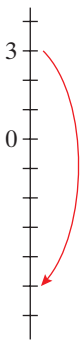
Negative numbers

Numbers less than zero are called **negative numbers**.

Example 1.11

The temperature at 4 p.m. is 3°C . By midnight it has fallen 8 degrees. What is the temperature at midnight?

Solution



Moving down 8 degrees goes to 5 below zero.
The answer is written -5°C and you say 'negative 5' or 'minus 5'.

A number line is very useful when working with negative numbers.

Notice that the further left, the smaller the number. For example, -2 is smaller than 1 .



Example 1.12

Start at -2 .

- (a) Add 5 (b) Add 2 (c) Subtract 4

Solution

Use the number line.

- (a) Count 5 to the right. The answer is 3.
(b) Count 2 to the right. The answer is 0.
(c) Count 4 to the left. The answer is -6 .

Exercise 1.5

- 1 Work out these.
- (a) -4 add 7 (b) -7 add 4 (c) 9 subtract 12
- 2 The temperature is -6°C . Find the new temperature after
- (a) a rise of 5°C . (b) a rise of 10°C . (c) a fall of 2°C .

Exercise continues ...

- 3 Find the difference in temperature between
 (a) 5°C and 21°C . (b) -5°C and 21°C . (c) -18°C and -4°C .
- 4 Arrange these numbers in order, smallest first.
 (a) 1, -3, 7, -8 (b) 0, -4, 5, -6 (c) 1, 2, -3, -4, 5, -6
- 5 An office building has 20 floors and three levels of underground car park. In the lift, the ground floor button is labelled 0. What should be the label on the button for the lowest car park level?

Challenge 1.4



Write the temperatures of the cities on the map in order, from coldest to warmest.



Challenge 1.5



Copy and complete the table.

Starting temperature	Movement	Final temperature
-3°	Up 5°	
7°	Down 10°	
-6°	Up 4°	
-4°		2°
3°		-8°
-7°		-3°
	Up 6°	0°
	Down 7°	-4°
	Down 3°	-6°

What you have learned

- An integer is a whole number
- A multiple is a result of multiplying one integer by another integer; a common multiple of two integers is a multiple of both of the integers
- An integer that will divide into a number exactly is called a factor of the number; a common factor of two numbers is an integer that will divide exactly into both numbers
- To round numbers to the nearest 10, you look at the digit in the units place; if it is less than 5 you leave the tens digit as it is; if it is 5 or more you add one to the tens digit; you replace the remaining digits with zeros
- You use similar methods to round to the nearest 100, 1000, ...
- To multiply by 10 you move the digits one place to the left and add a zero in the units place; to multiply by 100 you move the digits two places to the left and add zeros in the units and tens place; to multiply by 1000 you move the digits three places to the left and add zeros in the units, tens and hundreds place
- To divide by 10 you move the digits one place to the right; to divide by 100 you move the digits two places to the right; to divide by 1000 you move the digits three places to the right
- A square is a number multiplied by itself; you can write '5 squared' as 5^2 and it means 5×5
- You can write '2 cubed' as 2^3 and it means $2 \times 2 \times 2$
- You can write '3 to the power 4' as 3^4 and it means $3 \times 3 \times 3 \times 3$
- You can write 'the square root of 36' as $\sqrt{36}$; finding a square root is the reverse of finding a square so $\sqrt{36} = 6$
- You can find squares, cubes, other powers and square roots using your calculator
- Numbers less than zero are negative

Mixed exercise 1

1 Work out these.

(a) $59 + 73$

(b) $62 - 18$

(c) $456 - 187$

(d) 58×6

(e) 254×4

(f) $441 \div 7$

2 Jane bought six pens at 38p each and a notebook for 43p. Find the total cost in pence.

3 (a) List the multiples of 20 less than 125.

(b) List the multiples of 12 less than 125.

(c) List the common multiples of 12 and 20 less than 125.

Mixed exercise 1 continues ...

- 4** (a) List the factors of 12. (b) List the factors of 18.
 (c) List the factors of 30. (d) List the common factors of 12, 18 and 30.

5 Round

- (a) 5632 to the nearest 100. (b) 17 849 to the nearest 1000.
 (c) 273 490 to the nearest 1000. (d) 273 490 to the nearest 100.
 (e) 5 836 492 to the nearest million. (f) 3498 to the nearest 10.

6 Work out these.

- (a) 93×100 (b) 630×100 (c) 572×1000
 (d) $7800 \div 100$ (e) $6\,300\,000 \div 1000$ (f) 50×80
 (g) 70×300 (h) 47×30 (i) 58×600
 (j) 28×5000 (k) 456×70 (l) 732×400

7 Work out these. Show your working.

- (a) 63×28 (b) 83×57 (c) 256×38

8 In a sponsored walk 186 people each walked 45 km.
 What was the total distance they walked? Show your working.

9 Work out these.

- (a) 8^2 (b) 40^2 (c) 500^2 (d) 10^5 (e) 20^3

10 Use your calculator to work out these.

- (a) 29^2 (b) 12^3 (c) 53^2 (d) $\sqrt{484}$ (e) $\sqrt{5184}$

11 Copy and complete the table.

Temperature (°C)	5		-4	
10° warmer		7		
5° colder				-15