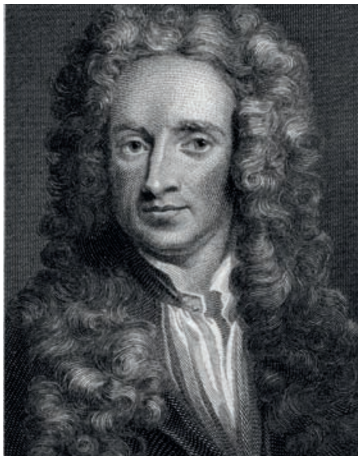
SECTION 1 The British



Isaac Newton (1642–1727)

Isaac Newton was born in Lincolnshire in 1642 and was probably the greatest scientist and mathematician ever to have lived. He was 22, and on leave from Cambridge University, when he began mathematical work on optics, dynamics, thermodynamics, acoustics and astronomy. He studied gravitation and the idea that white light is a mixture of all the rainbow's colours. He also designed the first reflecting telescope, the first reflecting microscope, and the sextant.

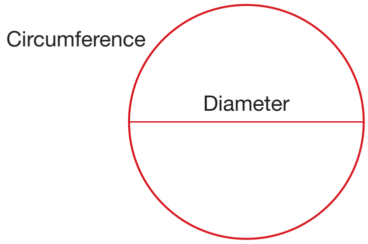
Newton is widely regarded as the 'Father of Calculus'. He discovered what is now called the Fundamental Theorem of Calculus, i.e. that integration and differentiation are each other's inverse operation. He applied calculus to solve many problems including finding areas, tangents, the lengths of curves and the maxima and minima of functions.

In 1687 Newton published *Philosophiae Naturalis Principia Mathematica*, one of the greatest scientific books ever written. The movement of the planets was not understood before Newton's Laws of Motion and the Law of Universal Gravitation. The idea that the Earth rotated about the Sun was introduced in ancient Greece, but Newton explained why this happens.

SECTION 2 Circumference and area of a circle

All circles are similar shapes. As a result, the ratio of their circumference to diameter is constant.

i.e.



$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant}$$

The constant is π(pi) which is 3.14 to 3 s.f.

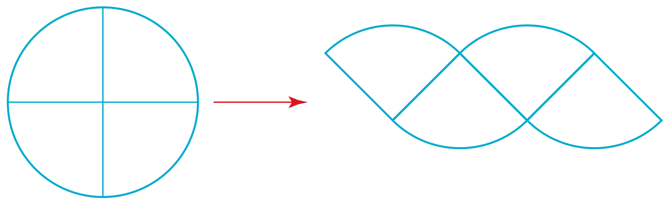
Therefore $\dfrac{C}{D} = \pi$

But the diameter is $2 \times$ radius, so the above equation can be written as $\dfrac{C}{2r} = \pi$

So the circumference of a circle, $C = 2\pi r$

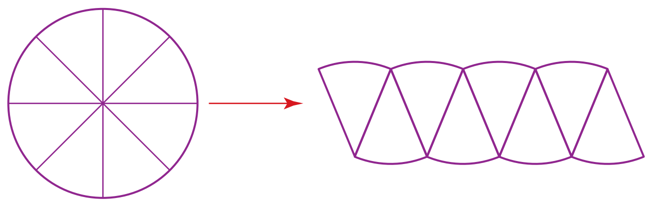
The area (A) of a circle can also be justified.

The diagram below shows a circle divided into four sectors. The sectors have then be rearranged and assembled as shown.

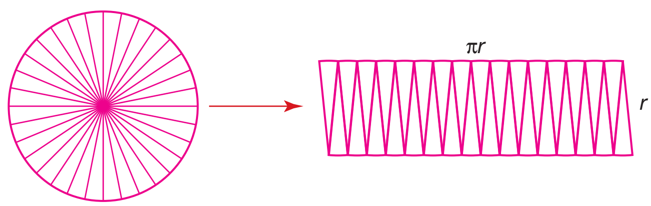


The total length of the curved edges is the same as the circumference of the circle.

If the circle is divided into eight sectors and each assembled as before, the diagram is:



As the number of sectors increases, the assembled shape begins to look more and more like a rectangle, as shown below with 32 sectors.

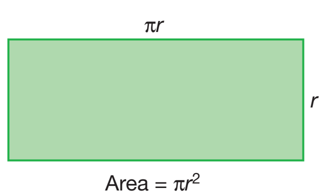


The top and bottom of the 'rectangle' is still equivalent to the circumference of the circle = $2\pi r$.

The top is therefore half the circumference = πr.

The height of the 'rectangle' is nearly equivalent to the radius of the circle.

With an infinite number of sectors, the circle can be rearranged to form a rectangle with a width πr and a height r.

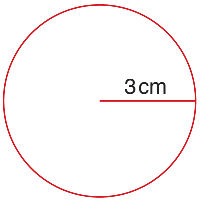


So the area of a circle, $A = \pi r^2$

Worked examples

NB: All diagrams are not drawn to scale.

a) Calculate the circumference of this circle, giving your answer to 3 s.f.

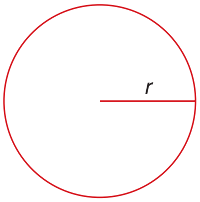


$C = 2\pi r$

$= 2\pi \times 3$

$= 18.8496$

The circumference is 18.8 cm.



b) If the circumference of this circle is 12 cm, calculate the radius, giving your answer to 3 s.f.

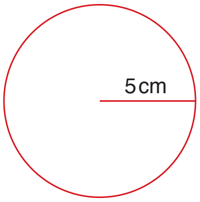
$C = 2\pi r$

$r = \dfrac{C}{2\pi}$

$r = \dfrac{12}{2\pi}$

$= 1.90986$

The radius is 1.91 cm.



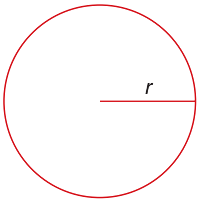
c) Calculate the area of this circle, giving your answer to 3 s.f.

$A = \pi r^2$

$= \pi \times 5^2$

$= 78.5398$

The area is 78.5 cm^2.



d) If the area of this circle is 34 cm^2, calculate the radius, giving your answer to 3 s.f.

$A = \pi r^2$

$r = \sqrt{\dfrac{A}{\pi}}$

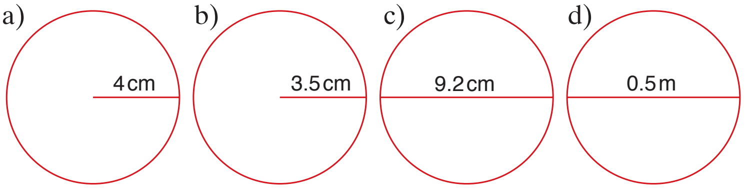
$r = \sqrt{\dfrac{34}{\pi}}$

$= 3.2898$

The radius is 3.29 cm.

Exercise 6.1

1. Calculate the circumference of each circle, giving your answer to 3. s.f.



2. Calculate the area of each of the circles in Q.1. Give your answers to 3 s.f.

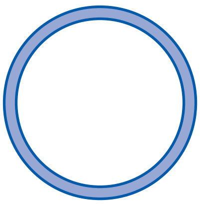
3. Calculate the radius of a circle when the circumference is:
   a) 15 cm b) π cm
   c) 4 m d) 8 mm

4. Calculate the diameter of a circle when the area is:
   a) $16\,cm^2$ b) $9\pi\,cm^2$
   c) $8.2\,m^2$ d) $14.6\,mm^2$

Exercise 6.2

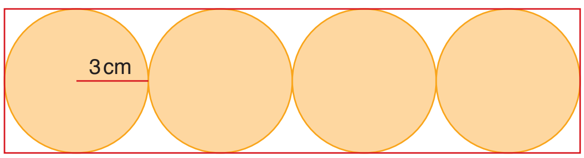
1. The wheel of a car has an outer radius of 25 cm. Calculate:
   i) how far the car has travelled after one complete turn of the wheel
   ii) how many times the wheel turns for a journey of 1 km.

2. If the wheel of a bicycle has a diameter of 60 cm, calculate how far a cyclist will have travelled after the wheel has rotated 100 times.



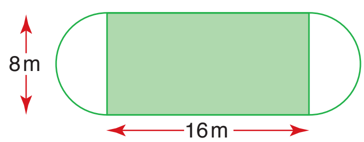
3. A circular ring has a cross-section as shown here. If the outer radius is 22 mm and the inner radius 20 mm, calculate the cross-sectional area of the ring.

4. Four circles are drawn in a line and enclosed by a rectangle as shown.



If the radius of each circle is 3 cm, calculate:
   a) the area of the rectangle
   b) the area of each circle
   c) the unshaded area within the rectangle.

5. A garden is made up of a rectangular patch of grass and two semicircular vegetable patches.

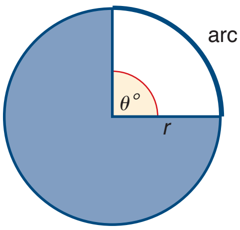


If the length and width of the rectangular patch are 16 m and 8 m respectively, calculate:
   a) the perimeter of the garden
   b) the total area of the garden.

SECTION 3

Arc length and area of a sector

NB: All diagrams are not drawn to scale.



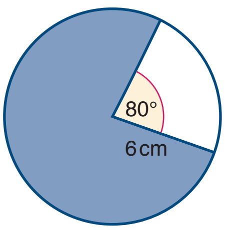
Arc length

An **arc** is part of the circumference of a circle between two radii. Its length is proportional to the size of the angle θ between the two radii. The length of the arc as a fraction of the circumference of the whole circle is therefore equal to the fraction that θ is of 360°.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

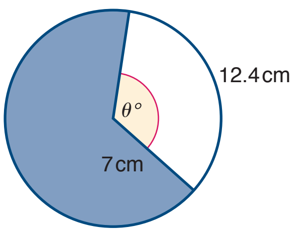
Worked examples

a) Find the length of the minor arc in the circle below. Give your answer to 1 d.p.



$$\text{Arc length} = \frac{80}{360} \times 2 \times \pi \times 6$$
$$= 8.4\,\text{cm}$$

b) In this circle, the length of the minor arc is 12.4 cm and the radius is 7 cm.



i) Calculate the angle $\theta°$.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$
$$12.4 = \frac{\theta}{360} \times 2 \times \pi \times 7$$
$$\frac{12.4 \times 360}{2 \times \pi \times 7} = \theta$$
$$\theta = 101.5 \text{ (1 d.p)}$$

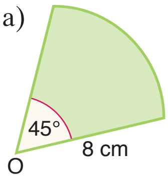
ii) Calculate the length of the major arc.

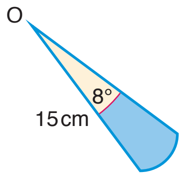
$$C = 2\pi r$$
$$= 2 \times \pi \times 7$$
$$= 44.0\,\text{cm (3 s.f.)}$$

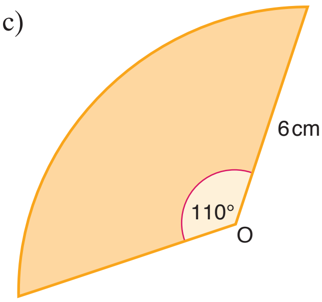
$$\text{Major arc} = \text{circumference} - \text{minor arc}$$
$$= (44.0 - 12.4)\,\text{cm}$$
$$= 31.6\,\text{cm}$$

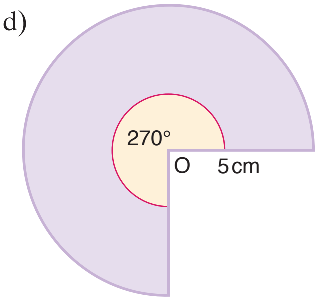
Exercise 6.3

1. For each of the following, give the length of the arc to 3 s.f. O is the centre of the circle.

a) 

b) 

c) 

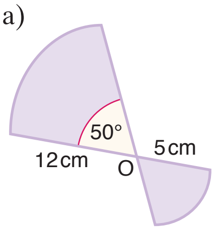
d) 

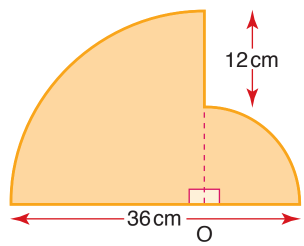
2. A sector is the region of a circle enclosed by two radii and an arc. Calculate the angle θ for each of the following sectors. The radius r and arc length a are given in each case.
   a) $r = 14\,\text{cm}$, $a = 8\,\text{cm}$
   b) $r = 4\,\text{cm}$, $a = 16\,\text{cm}$
   c) $r = 7.5\,\text{cm}$, $a = 7.5\,\text{cm}$
   d) $r = 6.8\,\text{cm}$, $a = 13.6\,\text{cm}$

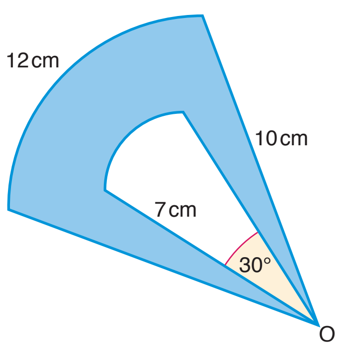
3. Calculate the radius r for each of the following sectors. The angle θ and arc length a are given in each case.
   a) $\theta = 75°$, $a = 16\,\text{cm}$
   b) $\theta = 300°$, $a = 24\,\text{cm}$
   c) $\theta = 20°$, $a = 6.5\,\text{cm}$
   d) $\theta = 243°$, $a = 17\,\text{cm}$

Exercise 6.4

1. Calculate the perimeter of these shapes.

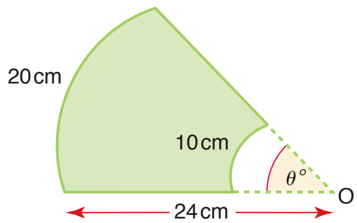
a) 

b) 



2. A shape is made from two sectors arranged in such a way that they share the same centre. The radius of the smaller sector is 7 cm and the radius of the larger sector is 10 cm.

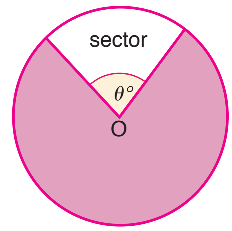
   If the angle at the centre of the smaller sector is 30° and the arc length of the larger sector is 12 cm, calculate:
   a) the arc length of the smaller sector
   b) the total perimeter of the two sectors
   c) the angle at the centre of the larger sector.



3. For the diagram on the left, calculate:
a) the radius of the smaller sector
b) the perimeter of the shape
c) the angle $\theta°$.

The area of a sector

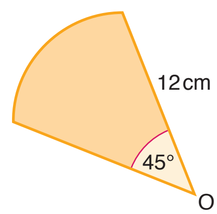
A **sector** is the region of a circle enclosed by two radii and an arc. Its area is proportional to the size of the angle $\theta°$ between the two radii. The area of the sector as a fraction of the area of the whole circle is therefore equal to the fraction that $\theta°$ is of 360°.



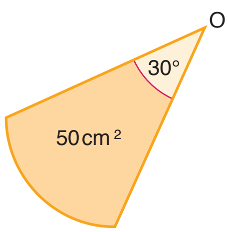
$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

Worked examples

a) Calculate the area of the sector (right), giving your answer to 1 d.p.



$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{45}{360} \times \pi \times 12^2$$
$$= 56.5\,\text{cm}^2$$



b) Calculate the radius of the sector (left), giving your answer to 3 s.f.

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$
$$50 = \frac{30}{360} \times \pi \times r^2$$
$$\frac{50 \times 360}{30\pi} = r^2$$
$$r = 13.8$$

The radius is 13.8 cm.

Exercise 6.5

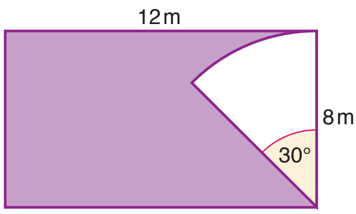
1. Calculate the area of each of the following sectors, using the values of the angles θ and radius r in each case.
a) $\theta = 60°$, $r = 8\,\text{cm}$
b) $\theta = 120°$, $r = 14\,\text{cm}$
c) $\theta = 2°$, $r = 18\,\text{cm}$
d) $\theta = 320°$, $r = 4\,\text{cm}$

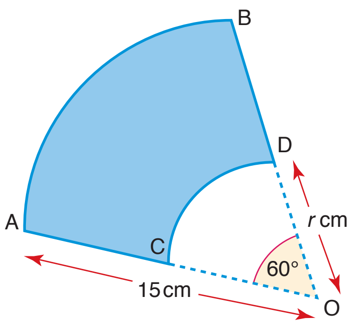
2. Calculate the radius for each of the following sectors, using the values of the angle θ and the area A in each case.
a) $\theta = 40°$, $A = 120\,\text{cm}^2$
b) $\theta = 12°$, $A = 42\,\text{cm}^2$
c) $\theta = 150°$, $A = 4\,\text{cm}^2$
d) $\theta = 300°$, $A = 400\,\text{cm}^2$

3. Calculate the value of the angle θ, to the nearest degree, or each of the following sectors, using the values of A and r in each case.
   a) $r = 12\,\text{cm}$, $A = 60\,\text{cm}^2$
   b) $r = 26\,\text{cm}$, $A = 0.02\,\text{m}^2$
   c) $r = 0.32\,\text{m}$, $A = 180\,\text{cm}^2$
   d) $r = 38\,\text{mm}$, $A = 16\,\text{cm}^2$

Exercise 6.6

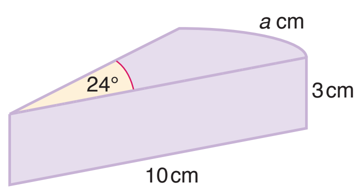
1. A rotating sprinkler is placed in one corner of a garden as shown. If it has a reach of 8 m and rotates through an angle of 30°, calculate the area of garden not being watered.

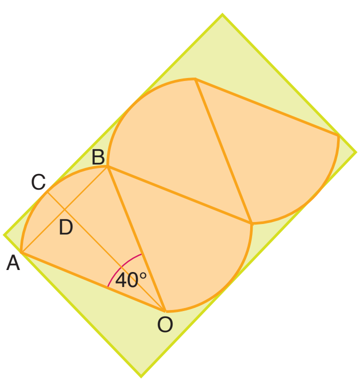




2. Two sectors AOB and COD share the same centre O. The area of AOB is three times the area of COD. Calculate:
   a) the area of sector AOB
   b) the area of sector COD
   c) the radius r cm of sector COD.

3. A circular cake is cut. One of the slices is shown. Calculate:
   a) the length a cm of the arc
   b) the total surface area of all the sides of the slice.





4. The diagram (left) shows a plan view of four tiles in the shape of sectors placed in the bottom of a box. C is the midpoint of the arc AB and intersects the chord AB at point D. If the length ADB is 8 cm and the length OB is 10 cm, calculate:
   a) the length OD
   b) the length CD
   c) the area of the sector AOB
   d) the length and width of the box
   e) the area of the base of the box not covered by the tiles.